D.A.V. INSTITUTIONS, CHHATTISGARH

PRACTICE PAPER-5 : 2023-24

CLASS – XII

SUBJECT- MATHEMATICS (041)

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

1. **All questions are compulsory.**
2. **The question paper has five sections. Section–A, Section-B, Section-C, Section-D and Section–E. There are 38 questions in the question paper.**
3. **Section–A has 18 MCQ questions and 2 Assertion- Reason based question of 1 marks each. Section–B has 5 Very Short Answer (VSA) type questions of 2 marks each, Section-C has 6 Short Answer (SA) type questions of 3 marks each, Section–D has 4 Long Answer (LA) type questions of 5 marks each and Section–E has 3 case based questions of 4 marks each.**
4. **There is no overall choice. However internal choice have been provided in some questions. Attempt only one of the alternatives in such questions.**
5. **Wherever necessary, neat and properly labelled diagram should be drawn.**

SECTION A(Multiple Choice Questions) **Each question carries 1 mark**
1. If $A = \begin{bmatrix} -a & b \\ c & a \end{bmatrix}$ and A^2 = I, then a) $a^2 + bc - 1 = 0$ b) $1 - a^2 + bc = 0$ c) $a^2 + bc + 1 = 0$ d) $a^2 - bc + 1 = 0$
2. If A is a square matrix of order 3 such that IAI = - 5 , then value of $\|-AA'\|$ is (a) 125 (b) - 125 (c) 25 (d) - 25
3. The Integrating factor of the differential equation $(1-y^2)\frac{dx}{dy} + yx = ay$ is a) $\frac{1}{y^2-1}$ b) $\frac{1}{\sqrt{y^2-1}}$ c) $\frac{1}{1-y^2}$ d) $\frac{1}{\sqrt{1-y^2}}$
4. For any square matrix A, AA^T is a (a) unit matrix (b) symmetric matrix (c) skew-symmetric matrix (d) diagonal matrix
5. If $A = \begin{vmatrix} 1 & 1 & -2 \\ \lambda & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$ then A^{-1} exist if a) $\lambda = 2$ b) $\lambda = 0$ c) $\lambda \neq 2$ d) $\lambda \neq 0$
6. The Differential coefficient of $\sec(\tan^{-1} x)$ w.r.t. x is (a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\frac{x}{1+x^2}$ (c) $x\sqrt{1+x^2}$ (d) $\frac{1}{\sqrt{1+x^2}}$
7. If A is a 3 x 3 matrix and $\|A\| = -2$ then value of $\|A(\text{adj}A)\|$ is (a) -2 (b) 2 (c) -8 (d) 8
8. P is a point on the line joining the points (0,5, -2) and B (3, -1,2) . If the x-coordinate of P is 6, then its z-coordinate is (a) 10 (b) 6 (c) -6 (d) -10

9. $\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

(a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

(c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (d) $-\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

10. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

11.The value of $(\hat{i}\times\hat{j}).\hat{k}+(\hat{j}\times\hat{k}).\hat{i}+(\hat{k}\times\hat{i}).\hat{j}$ **is**

(a) 0 (b) –1 (c) 1 (d) 3

12. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).Let F = 4*x* + 6*y* be the objective function. The Minimum value of F occurs at

(a) (0, 2) only (b) (3, 0) only

(c) the mid point of the line sgment joining the points (0, 2) and (3, 0) only

(d) any point on the line segment joining the points (0, 2) and (3, 0).

13. The projection of $\vec{a}$ on $\vec{b}$, if $\vec{a}.\vec{b}$ =8 and $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$.

(a) $\frac{8}{7}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{5}$

14. If $|\vec{a}+\vec{b}|$= 60, $|\vec{a}-\vec{b}|$= 40 and $|\vec{a}|$= 22 then $|\vec{b}|$ =

(a) 36 (b) 22/60 (c) 46 (d) None of these

15. For what value of k the function $f(x) = \begin{cases} 2x+1 & x<2 \\ k & x=2 \\ 3x-1 & x>2 \end{cases}$ is continuous at x = 2 ,

a) Any real value b) No real value c) 5 d) 1/5

16. Given two independent events A and B such that P(A) =0.3, P(B) = 0.6 and P($A'\cap B$) is

a) 0.42(b) 0.18 (c) 0.28 (d) 0.12

17.The corner points of the feasible region determined by the following system of linear inequalities: 2x + y ≤ 10, x + 3y ≤ 15, x, y ≥ 0 are (0,0), (5,0), (3,4), (0,5). Let Z= px + qy, where p,q> 0. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is

a) p = q b) p = 2q c) p = 3q d) q = 3p

18. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

a) 2 b) 3/4 c) 0 d) - 2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) Both A and R are false.

19. Assertion(A) : The pair of lines given by $\vec{r} = \hat{i}-\hat{j}+\lambda(2\hat{i}+\hat{k})$ and $\vec{r} = 2\hat{i}-\hat{k}+\mu(\hat{i}+\hat{j}-\hat{k})$ intersect .

Reason(R) :Two lines intersect each other, if they are not parallel and shortest distance = 0.

20. Assertion (A) : The value of expression $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) + \tan^{-1}(1) + \sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{\pi}{4}$

Reason (R) : Principal value branch of $\sin^{-1}(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $\sec^{-1}(x)$ is $[0, \pi]$

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. Write the simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

Or

Show that $f : N \to N$, given by $f(n) = \begin{cases} n+1, & if\ n\ is\ odd \\ n-1, & if\ n\ is\ even \end{cases}$ is a bijection.

22. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

23. The volume of a cube is increasing at the rate of 9 cubic cm per sec .How fast is the surface area increasing when the length of an edge is 10 cm.

24. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

OR

If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

25. Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other .

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Evaluate $\int \frac{x^2}{x^2+6x+12}\, dx$

OR

Evaluate $\int \frac{\sin^{-1} x}{x^2}\, dx$

27. Two cards are drawn simultaneously from a well shuffled pack of 52 playing cards. Find the mean of the number of aces drawn.

Or

Three friends go for coffee. They decide who will pay the bill, by each tossing a coin and then letting the "odd person" pay. There is no odd person if all three tosses produce the same result. If there is no odd person in the first round, they make a second round of tosses and they continue to do so until there is an odd person. What is the probability that exactly three rounds of tosses are made?

28. Evaluate $\int_0^{\pi/2} \log(\tan x)\, dx$

29. Solve following linear programming problem graphically
Maximise Z= 400 x + 300 y subject to the constraints $x + y \leq 200$, $x \leq 40$, $x \geq 20$, $y \geq 0$

30. Evaluate $\int \frac{dx}{x\,(x^5+1)}$

31. Solve the differential equation $\cos x \frac{dy}{dx} + y = \sin x$

OR

Solve the differential equation $Sec^2 y(1+x^2)\,dy + 2x \tan y\, dx = 0$; $y = \pi/4$, $x = 1$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Find the equation of line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ & $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

Or

Find the equation of the perpendicular from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also find the foot of perpendicular and length of perpendicular .

33. Find the product of the matrices A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the equations x + y + 2z = 1, 3 x + 2 y + z = 7 and 2 x + y +3z = 2.

34. Using the method of integration , find area of the region bounded by lines 2x +y = 4 , 3x -2y = 6 and x -3y +5 = 0 .

35. For the Power set of all subsets of a non empty set , a relation A R B is defined if and only if A $\subset$ B. Is R an equivalence relation on the power set ?

Or

Show that the relation R in the set NxN defined by (a, b)R(c, d) if $a^2+d^2= b^2+c^2 \forall$ a, b, c, d $\in$ N, is an equivalence relation.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts.First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub -parts of 2 marks each.)

36.**Case-Study 1 :**Read the following passage and answer the questions given below.
The reliability of a COVID 19 test is specified as follows :
Of people having COVID 19, 90% of the test detect the disease but 10% go undetected.
Of people free of COVID 19, 99% of the test are judged COVID 19 negative but 1% are diagnosed as showing COVID 19 positive.
From a large population of which only 0.1% have COVID 19, one person is selected at random, given the COVID19 test, and the pathologist reports him/her as COVID 19 positive.

(i) What is the probability of the 'person to be tested as COVID19 positive' given that 'he is actually having COVID 19'and What is the probability of the 'person to be tested as COVID 19 positive' given that 'he is actually not having COVID 19' ?
(ii) What is the probability that the 'person is actually not having COVID 19'?
(iii) What is the probability that the 'person is actually having COVID 19' given that 'he is tested as COVID19 positive'?

Or

(iii) What is the probability that the 'person selected will be diagnosed as COVID 19 positive'?

37. **Case-Study 2 :**Read the following passage and answer the questions given below.

In a street two lamp posts are 300 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source).

The combined light intensity is the sum of the two light intensities coming from both lamp posts.

(i) If you are in between the lamp posts, at distance x feet from the stronger light, then find the formula for the combined light intensity coming from both lamp posts as function of x .
(ii) What will be the maximum value and minimum of x ?
(iii) If I(x) denote the combined light intensity, then find the value of x for which I(x) will be minimum ?

Or

(iv) Find the distance of the darkest spot between the two lights ?

38. **Case-Study 3 :**Read the following passage and answer the questions given below.

A real estate company is going to build a new apartment complex. The land they have purchased can hold at most 5000 apartments. Also, if they make x apartments, then the maintenance costs for the building, landscaping etc., would
be as follows:
Fixed cost = Rs 40,00,000
Variable cost =Rs $(140x – 0.04x^2)$

CamScanner

If C(x) denote the maintenance cost function, then $C(x) = 40,00,000 + 140x - 0.04x^2$

(i) Find the intervals in which the function C(x) is strictly increasing/strictly decreasing.

(ii) Find the points of local maximum/local minimum, if any, in the interval (0, 5000) as well as the points of absolute maximum/absolute minimum in the interval [0, 5000]. Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.