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Please check that this question paper contains 38 Questions and has 4 Printed pages.

D.A.V. INSTITUTIONS, CHHATTISGARH

Practice Paper-10

CLASS - XII

SUBJECT: - MATHEMATICS

Time Allowed: 3 Hours

Maximum Marks:- 80

GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section -A

(Multiple Choice Questions)

Each question carries 1 mark

1. If $A = (a_{ij})_{2 \times 2}$ is a matrix where $a_{ij} = e^{2ix} \sin jx$, then the value of the element a_{12} is
(a) $e^x \sin x$ (b) $e^{2x} \sin x$ (c) $e^x \sin 2x$ (d) $e^{2x} \sin 2x$
2. If $5A$ is a square matrix of order 3 and $|5A| = P|A|$, then value of P is
(a) 5 (b) 25 (c) 1 (d) 125
3. The area of triangle with vertices A, B and C is given by
(a) $\vec{AB} \times \vec{AC}$ (b) $\frac{1}{2} \vec{AB} \times \vec{AC}$ (c) $\frac{1}{4} \vec{AB} \times \vec{AC}$ (d) $\frac{1}{8} \vec{AC} \times \vec{AB}$
4. The function $f(x) = \frac{5-x^2}{9x-x^3}$ is
(a) Discontinuous at only one point (b) Discontinuous at exactly two points
(c) Discontinuous at exactly three points (d) Discontinuous at no any point
5. The lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(6\hat{i} + 9\hat{j} - 18\hat{k})$ are
(a) Coincident (b) skew (c) Intersecting (d) Parallel
6. The integrating factor of $x \log x \frac{dy}{dx} + y = 2 \log x$ is
(a) $\log x$ (b) $2 \log x$ (c) $-\log x$ (d) $x \log x$
7. The corner points of the feasible region determined by the system of linear inequalities are (0,0), (4,0) (2,4) and (0,5). If the maximum value of $Z = ax + by$, where $a, b > 0$ occurs at (2,4) and (4,0) then
(a) $a = 2b$ (b) $2a = b$ (c) $a = b$ (d) $3a = b$
8. If $|\vec{a}| = 10$, $|\vec{b}| = 2$, $|\vec{a} \times \vec{b}| = 16$ Then value of $\vec{a} \cdot \vec{b}$ is
(a) ± 20 (b) ± 12 (c) ± 16 (d) ± 24
9. The value of $\int_0^{1.5} [x] dx$ is equal to
(a) 0 (b) 0.5 (c) 1 (d) 2
10. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$ Then value of $|AB|$ is
(a) -30 (b) 30 (c) 35 (d) -35
11. The solution set of the inequality $3x + 5y < 4$ is
(a) an open half plane not containing the origin. (b) an open half plane containing the origin.

- © The whole XY-plane not containing the line $3x + 5y = 4$ (d) A closed half plane containing the origin.
12. If \vec{a} and \vec{b} are two unit vectors inclined to x-axis at angles 30° and 120° respectively, then value of $|\vec{a} + \vec{b}|$ is
 (a) 2 (b) $\sqrt{2}$ (c) 1 (d) 0
13. Given that A is a square matrix of order 3 and $|A| = -2$, then $|adj(2A)|$ is equal to
 (a) -2^6 (b) 4 (c) -2^8 (d) 2^8
14. If A and B are two events such that $P(A/B) = P$, $P(A) = P$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$, then P is equal to
 (a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
15. The solution of differential equation $2x \frac{dy}{dx} - y = 9$ represents a family of
 (a) Straight line (b) circles (c) parabolas (d) ellipses
16. The value of λ for which two vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular is
 (a) 2 (b) 4 (c) 6 (d) 8
17. The derivative of $\cot^{-1}(e^x)$ w.r.x. at the point $x = 0$ is
 (a) 0 (b) 1 (c) $1/2$ (d) $-1/2$
18. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) 0

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
 (c) (A) is true but (R) is false.
 (d) (A) is false but (R) is true.
19. Assertion(A) : If $y = \log(\sin e^x)$ then $\frac{dy}{dx} = e^x \cot e^x$
 Reason (R) : $y = f(g(x))$; $\frac{dy}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$; $\frac{d}{dx}(\log x) = \frac{1}{x}$
20. Assertion(A) : Relation $R = \{ (1,1), (1,3), (3,1), (3,3), (3,5) \}$ defined on the set $A = \{ 1,2,3 \}$ is symmetric.
 Reason(R) : A relation R is said to be symmetric if $(a,b) \in R \rightarrow (b,a) \in R$.

Section -B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the value of $\sin^{-1}(\cos(\frac{33\pi}{5}))$
22. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R
 OR
 The total cost $C(x)$ associated with provision of free mid-day meals to x students of a school in primary classes is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$, find the marginal cost of food for 300 students.
23. If $f(x) = \frac{1}{4x^2 + 2x + 1}$; $x \in R$, then find the maximum value of $f(x)$.
 OR
 Show that the function $f(x) = x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value.
24. Evaluate : $\int_0^{\frac{\pi}{2}} e^x (\sin x - \cos x) dx$

25. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 + x$, find the critical points of $f(x)$ if any, If no write the reason.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate : $\int \tan x \cdot \tan 2x \cdot \tan 3x \, dx$.

27. Four bad oranges are mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find mean of the distribution.

28. Evaluate : $\int_1^5 \{ |x - 1| + |x - 2| + |x - 3| \} dx$

OR

Evaluate : $\int x \cdot (\log x)^2 dx$.

29. Solve the differential equation : $(1 + e^{x/y}) dx + e^{x/y} (1 - \frac{x}{y}) dy = 0$.

OR

Solve the differential equation : $(x + 2y^2) \frac{dy}{dx} = y$, given that when $x = 2$, $y = 1$.

30. Solve the linear programming problem graphically: Maximize $Z = 2x + 4y$ subject to constraints $2x + 3y \leq 60$, $x + y \geq 15$, $x \leq y$; $x, y \geq 0$.

31. If $y = (\cos x)^{\log x} + (\log x)^x$; find $\frac{dy}{dx}$.

OR

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by line $x = a$. Find the value of a .

33. Consider $f: \mathbb{R} - \{-\frac{4}{3}\} \rightarrow \mathbb{R} - \{\frac{4}{3}\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is injective as well as surjective.

OR

Show that the relation R in the set of $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other

But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

34. Solve the system of equations by matrix method : $2x - 3y + z = -1$; $x - 2y + 3z = 6$; $-3y + 2z = 0$. Hence find the value of $x + y + z$.

35. Find the coordinates of the image of the point $(1, 6, 3)$ with respect to the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$; Also find the distance of the image from origin.

OR

There are two strings in space, they are represented by separate straight lines as follows :

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$

1) Are these strings parallel to each other? 2) Find the shortest distance between these strings.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts.

The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively.

The third case study question has two sub parts of 2 marks each.)

36. **Case-Study-1:** In answering a question on multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses.

Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1 , E_2 and E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

Based on above information, answer the following questions:



- (a) What is the value of $P(E_1)$? (1)
- (b) Find the value of $P(E/E_1)$ (1)
- (c) Find the value of $\sum_{k=1}^{k=2} P(E/E_k) P(E_k)$

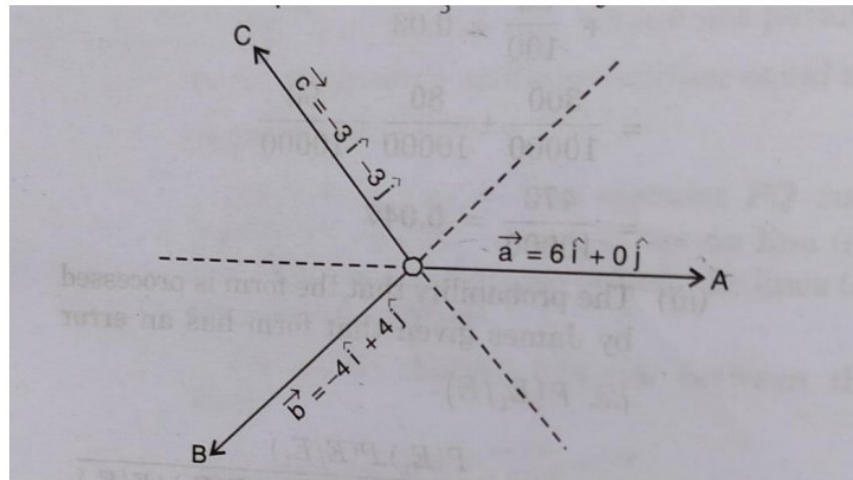
OR

What is the probability that the student knows the answer given that he answered it correctly. (2)

37. Case-Study-2: Read the following passage and answer the questions given below:

Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area. Team A pulls with $F_1 = 6\hat{i} + 0\hat{j}$ KN

Team B pulls with $F_2 = -4\hat{i} + 4\hat{j}$ KN ; Team C pulls with $F_3 = -3\hat{i} - 3\hat{j}$ KN.



Based on the above information, answer the following questions:-

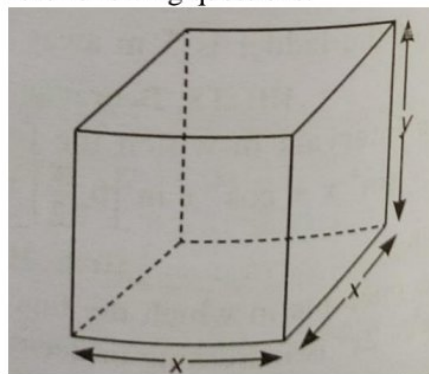
- (a) What is the magnitude of the force of team A ? (1)
- (b) Which team will win the game ? (1)
- (c) Find the magnitude of the resulting force exerted by the teams .

OR

In what direction is the ring getting pulled ? (2)

38. Case-Study3: A given quantity of metal sheet is to be cast into an open tank with a square base and vertical sides as shown. Where x be the side of base square and y be the vertical side.

Based on above information, answer the following questions:



- (a) If V represents the volume of tank, find the relation between V , x and y and find total area of tank as a function of x . (2)
- (b) Find x , when the total surface area of tank is minimum. (2)