D.A.V. INSTITUTIONS, CHHATTISGARH

PRACTICE PAPER-4 : 2023-24

CLASS – XII

SUBJECT- MATHEMATICS (041)

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

1. **All questions are compulsory.**
2. **The question paper has five sections. Section–A, Section-B, Section-C, Section-D and Section–E. There are 38 questions in the question paper.**
3. **Section–A has 18 MCQ questions and 2 Assertion- Reason based question of 1 marks each. Section–B has 5 Very Short Answer (VSA) type questions of 2 marks each, Section-C has 6 Short Answer (SA) type questions of 3 marks each, Section–D has 4 Long Answer (LA) type questions of 5 marks each and Section–E has 3 case based questions of 4 marks each.**
4. **There is no overall choice. However internal choice have been provided in some questions. Attempt only one of the alternatives in such questions.**
5. **Wherever necessary, neat and properly labelled diagram should be drawn.**

SECTION – A

(Multiple Choice Questions)

(Each question carries 1 mark)

Q1. If order and degree of the differential equation $\left(\frac{dx}{dy}\right)^4 + 3x\frac{d^2y}{dx^2} = 0$ is m and n respectively, then m – n is equal to:

(a) 1 (b) 2

(c) 3 (d) 0

Q2. The angle between the unit vectors $\hat{a}$ and $\hat{b}$, given that $|\hat{a} + \hat{b}| = 1$, is:

(a) π/3 (b) 2π/3

(c) –π/3 (d) π/2

Q3. The function f: R→ R given by f(x) = $\cos x \ \forall \ x \in R$, is:

(a) one-one (b) not one-one

(c) bijective (d) None of these

Q4. If f(x) = x$|x|$, then f'(x) is equal to:

(a) x^2 (b) $2x$

(c) $2|x|$ (d) 1

Q5. The value of x dy + (x-1)dx = 0 is …

(a) y = logx – x + C (b) y = logx + x + C

(c) y = -logx + C (d) – logx – x + C

Q6. If f(x) = 2 $|x| + 3|\sin x| + 6$, then the right hand derivative of f(x) at x = 0, is

(a) 1 (b) 5

(c) 3 (d) 4

Q7. Let R = {(a, a^3) : a is the prime number less than 5} be a relation , then the range of R is:

(a) {2,3} (b) {8,27}

(c) {2} (d) {2,3,8,27}

Q8. If a matrix A is both symmetric and skew symmetric, then A is

(a) null matrix (b) identity matrix

(c) diagonal matrix (d) none of these

Q9. The value of tan $\left[\frac{1}{2}\, cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is

(a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3-\sqrt{5}}{2}$

(c) $\frac{-3+\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$

Q10. If A $\equiv (2,3,1) and\ B\ \equiv (5,4,2)$, then the direction ratios of $\underset{AB}{\rightarrow}$ are:

(a) -3,1,1 (b) 3,1,1

(c) 5,4,2 (d) 3,0,1

Q11. A balloon which always remains spherical has a variable diameter = $\frac{3}{2}(2x+1)$. Then, find the rate of change of its volume with respect to x.

(a) $\frac{27\pi}{8}(2x+1)^2$ (b) $\frac{27\pi}{8}(2x+1)^2$

(b) (c) $\frac{27\pi}{16}(2x+1)^2$ (d) $\frac{27\pi}{8}(2x+1)^3$

Q12. The value of $\int sin^5\frac{x}{2} \cdot cos\frac{x}{2}dx$ is

(a) $\frac{1}{3}\left(sin^6\frac{x}{2}\right)+C$ (b) $\left(sin^6\frac{x}{2}\right)+C$

(c) $\frac{1}{3}\left(cos^6\frac{x}{2}\right)+C$ (d) $\left(cos^6\frac{x}{2}\right)+C$

Q13. A right circular triangle which is open at the top and has given surface area, will have the greatest volume, if its height H and radius r are related by

(a) 2h = r (b) h = 4r

(c) h = 2r (d) h = r

Q14. The value of $tan^{-1}\left\{tan\frac{15\pi}{4}\right\}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) - $\frac{\pi}{4}$ (d) π

Q15. If $[x]$ denotes the greater integer function, then $\int_0^{\frac{3}{2}}[x^2]\,dx$ is equal to

(a) $\sqrt{2}-2$ (b) $2-\sqrt{2}$ (c) $\sqrt{2}$ (d) $\sqrt{2}+2$

Q16. If A is symmetric matrix, then B'AB is

(a) Symmetric matrix (b) skew symmetric matrix (c) scalar matrix (d) none of the above

Q17.The direction ratios of the line segment joining P(x_1, y_1, z_1) and Q(x_2, y_2, z_2):

(a) $(x_1-x_2, y_1-y_2, z_1-z_2$ (b) $x_2-x_1, y_2-y_1, z_2-z_1$

(c) Both (a) and (b) (d) none of the above

Q18. The area of the triangle whose two sides are represented by the vectors $2\hat{\imath}\ and -3\hat{\jmath}$ is:

(a) 6 sq unit (b) 3 sq unit (c) 2 sq unit (d) 1 sq unit

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R).Choose the correct answer out of the following choices.

a) Both (A) and (R) are true and (R) is the correct explaination of (A).

b) Both (A) and (R) are true and (R) is not the correct explaination of (A).

c) (A) is true but (R) is false.

d) (A) is false but (R) is true.

Q19.Assertion (A): If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, thenn\ A + B = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

Reason(R): Two different matrices can be added only if they are of same order.

Q20.Assertion (A): Let f : R→ R be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 are $\pm$ 4.

Reason(R): The function f : A→ B is called a one-one function, if distinct elements of A have distinct images in B.

SECTION-B

(This section comprises of very short answer type questions (VSA) of 2 marks each.)

Q21. Find $|\vec{a} \text{ x } \vec{b}\,|, if\ \vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

And $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$

Q22. Find the area of the region bounded by the curve y = cos x between $\frac{\pi}{2}\ to\ \frac{3\pi}{2}$

Q23. Evaluate $\int \frac{1}{\sqrt{9+8x-x^2}} dx$

(OR)

Find $\int \frac{3-5\,sinx}{cos^2x} dx$

Q24. Show that function $x + \frac{1}{x}$ is increasing for x >1.

Q25. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then show that (A-2I)(A-3I)=O.

(OR)

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then find $A^2 - 4A + I$.

SECTION-C

(This section comprises of short type questions(SA) of 3 marks each)

Q26. Find the value of c, for which the vectors $\vec{a} = (c\ log_2 x)\hat{\imath} - 6\hat{\jmath} + 3\hat{k}\ and$

$\vec{b} = (log_2 x)\hat{\imath} + 2\hat{\jmath} + (2clog_2 x)\hat{k}$ make an obtuse angle for any $x \in (0, \infty)$.

Q27. Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)\, e^x dx = 0, given\ that\ y = 1, when\ x = 0.$

(OR)

Solve the following differential equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$.

Q28. If y = x^x, then prove that

$$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

(OR)

If y = $\frac{sin^{-1}x}{\sqrt{1-x^2}}$, then show that (1-$x^2$)$\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

Q29. Show that the function $f: R \to R$ defined by $f(x) = \frac{3x-1}{2}, x \in R$ is one-one and onto functions.

Q30 If x=a(2t- sint) and y = a(1- cost), then find $\frac{dy}{dx}$, when $\theta = \frac{\pi}{6}$.

Q31. Evaluate $\int_0^{\pi/2}(\sqrt{\tan x} + \sqrt{\cot x})\, dx$.

(OR)

Evaluate $\int_0^{\pi/2} e^x\left(\frac{1+sinx}{1+cosx}\right) dx$.

SECTION-D

(This section comprises of Long-Answer type questions (LA) of 5 marks each.)

Q32. Find $|\hat{a} - \hat{b}|$, if two vectors â and b̂ are such that $|\hat{a}| = 2, |\hat{b}| = 3\ and\ \hat{a}.\hat{b} = 4$.

(OR)

Find a unit vector perpendicular to both the vectors $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $-2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

Q33. Find the equation of line joining P(11,7) and Q(5,5) using determinants. Also, find the value of k, if R(-1,k) is the point such that area of ΔPQR is 9 sq m.

Q 34. Maximize Z = 8x + 9y subject to the constraints given below

$2x + 3y \le 6, 3x - 2y \le 6, y \le 1, x, y \ge 0.$

(OR)

Solve minimize Z = 5x + 7y

Subject to constraints

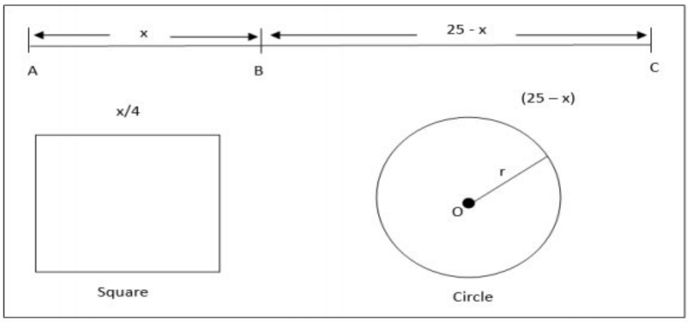
$2x + y \geq 8; x + 2y \geq 10, x, y \geq 0.$

Q 35. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ above x-axis.

SECTION- E

(This section comprises of 3 case-study/passage based questions of 4 marks each with sub-parts. The first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. The third case study question has two parts of 2 marks each.)

Q36. A person has a steel wire of length 25m he cut it into two pieces. Let x (m) and (25-x)m and bent x (m) in the shape of a square and (25-x) m in the shape of a circle as shown in the figure.



Based on the above information, answer the following questions.

(i) Find the combined area of square and circle as a function of x.

(ii) Find the length of two pieces.

Q 37. Integration is the process of finding the antiderivative of a function. In this process, we are provided with the derivative of a function and asked to find out the function (i.e primitive) integration is the inverse process of differentiation.

Let f(x) be a function of x. If there is a function g(x), such that $\frac{d}{dx}[g(x)] = f(x)$, then g(x) is called an integral of f (x) w.r.t. x and is denoted by $\int f(x)dx = g(x) + C$, where C is constant of integration.

Based on the above information, answer the following questions.

(i) $\int (3x+4)^3 dx \text{ is equal to}$

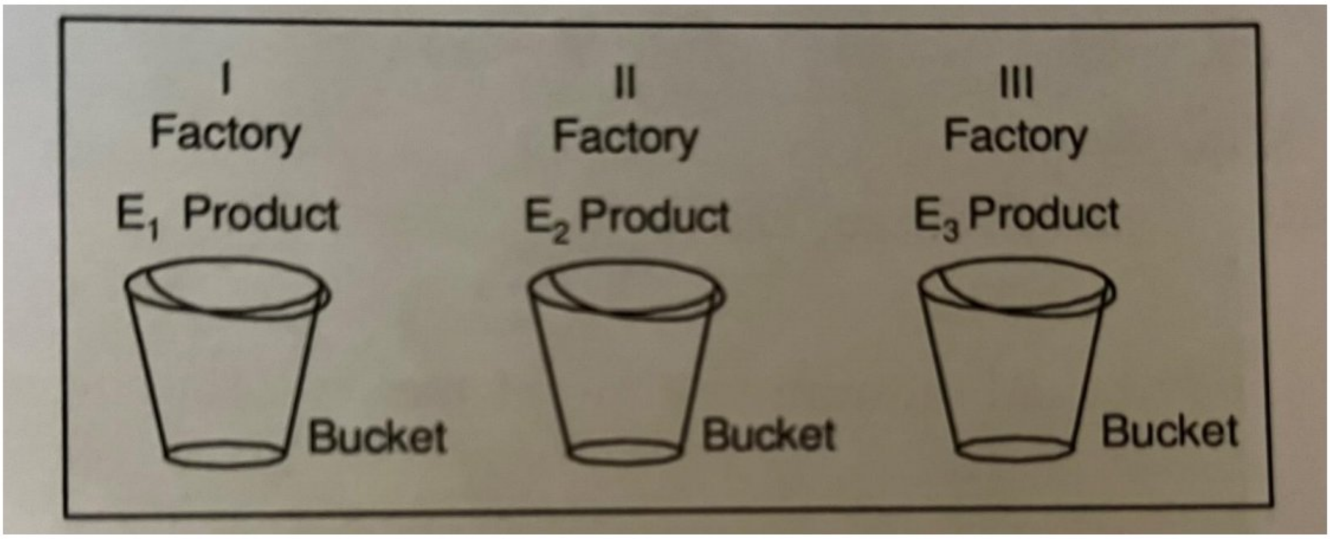
(ii) $\int \frac{(x+1)^2}{x(x^2+1)} dx \text{ is equal to}$

(iii) $\int \sin^2 x dx \text{ is equal to}$

OR

$\int tan^2 xdx$ *is equal to*

Q 38. Three machines E_1, E_2, E_3 in a certain factory produced 50%, 25%, 25% respectively of total output of buckets. 4% buckets produced by machines E_1 and E_2 (each are defective while 5% bucket produced by machine E_3 are defective.



Let A be the event that the produced bucket is defective then on the basis of above information, answer the following:

(i) Using concept of law of total probability, find the probability that manufactured bucket is defective taking from the lot.

(ii) Using Baye's theorem, find the probability that produced bucket is defective and it is manufactured by machine E_1.
