Roll No. | | | | | | | |

DAV PUBLIC SCHOOLS, ODISHA ZONE

HALF YEARLY EXAMINATION, 2023-24

- Please check that this question paper contains 6 printed pages.
- Check that this question paper contains 38 questions.
- Write down the Serial number of the question in the left side of the margin before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed 15 minutes prior to the commencement of the examination. The students will read the question paper only and will not write any answer on the answer script during this period.

CLASS - XI

SUB: MATHEMATICS (Code - 041)

Time: 3 Hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. **All** questions are **compulsory.**

(ii) This Question paper is divided into **five** sections−**A, B, C, D** and **E**.

(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs**)** and questions number **19** and **20** are Assertion-Reason based questions of 1 mark each.

(iv) In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.

(vii) In **Section E,** Questions no. **36** to **38** are case study based questions, carrying **4** marks each.

(viii)There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(i*x*) Use of calculators is **not** allowed.

SECTION-A

This section comprises multiple choice questions (MCQs) of 1 mark each**.**

1. For disjoint sets A and B, n (A) = 3, n (B) = 5, then n (A∪B) is

(a) 3 (b) 8 (c) 5 (d) 2

2. For sets A and B, n (A∪ B) $= 40$, n$(A - B) = 12$, n$(A' \cap B) = 20$, then n$(A \cap B)$ is

(a) 18 (b) 28 (c) 20 (d) 8

3. Let the set A = { x: x is a letter in the word "INTELLIGENT"}, then the number of non-empty proper subsets of A is

(a) 2^{11} (b) 2^{11}-1 (c) 64 (d) 62

4. Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

(a) 7, 6 (b) 3, 6 (c) 6, 3 (d) 8, 7

5. If $f(x) = \sqrt{36 - x^2}$, then the range of f (x) is

(a) $(-6, 6)$ (b) (0, 6) (c) $[-6, 6]$ (d) [0, 6]

6. The value of $\frac{1+tan^2 15^\circ}{2tan 15^\circ}$ is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{\sqrt{3}}{2}$

7. If $\tan\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{3}$, then the value of $(\alpha + \beta)$ is equal to

(a) $\frac{\pi}{6}$ (b) π (c) 0 (d) $\frac{\pi}{4}$

8. The Radian measure of $40°20'$ is equal to

(a) $\frac{121\pi}{504}$radian (b) $\frac{121\pi}{540}$radian (c) $\frac{121\pi}{3}$ radian (d) $\frac{120\pi}{540}$radian

9. If $3(7 + 7i) + i(7 + 7i) = a + ib$ then $\frac{b}{a} =$

(a) 2 (b) 1 (c) 3 (d) -1

10. The number of non-zero integral solution(s) of $|1 - i|^x = 2^x$ is

(a) 1 (b) 2 (c) 0 (d) 3

11. If $z_1 = -3+4i$, $z_2 = 4-3i$, then $\left|\frac{z_1}{z_2}\right|$ is

(a) 2 (b) 5 (c) 10 (d) 1

12. The set of values of x for which $-8 \leq 5x - 3 < 7$ and $4 - 2x < 0$, is

(a) ϕ (b) R (c) R – {0} (d) (2, 3)

13. If $|x + 2| \leq 9$, then

(a) $x \in (-7, 11)$ (b) $x \in [-11, 7]$

(c) $x \in (-\infty, -7) \cup (11, \infty)$ (d) $x \in (-\infty, -7) \cup (7, \infty)$

14. The length of a rectangle is three times its breadth (x). If the minimum perimeter of the rectangle is 160cm, then

(a) $x > 20$ (b) $x \geq 20$ (c) $x < 20$ (d) $x \leq 20$

15. The number of sides of a polygon having 44 diagonals is

(a) 8 (b) 9 (c) 10 (d) 11

16. If the 17^{th} and 18^{th} terms in the expansion of $(2 + a)^{50}$ are equal, then value of a is

(a) 1 (b) 15 (c) 3 (d) 2

17. The 4^{th} term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$ is

(a) $\frac{674}{x}$ (b) $\frac{670}{x^3}$ (c) $\frac{672}{x^3}$ (d) $-\frac{670}{x^3}$

18. The number of terms in the expansion of $(1 - 3x + 3x^2 - x^3)^9$ is equal to

(a) 10 (b) 27 (c) 28 (d) 9

Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. Assertion (A): Domain of R = $\{(x, y) : x, y \in W, x^2 + y^2 = 100\}$ is $\{0, 6, 8, 10\}$.

Reason (R): Domain of R = $\{x: x \in A, (x, y) \in R$, for some $y \in B\}$, where R is a relation from A to B.

20. Assertion (A): If the coefficient of x^2 in the expansion of $(1 + x)^m$ is 6, then m = 4.

Reason (R): The coefficient of x^q in the expansion of $(1 - x)^p$ is p_{C_q} .

SECTION–B

This Section Comprises very short answer (VSA) type questions of 2 marks each.

21. Show that $(A \cup B) = (A \cap B)$ implies $A = B$.

OR

For any sets A and B, Show that $A = (A \cap B) \cup (A - B)$.

22. If A = {5,6,7,8,9}, B = {2,4,6,8,10,12} and C = {3,6,9.12}, then verify that A – (B∪C) = (A – B) ∩ (A – C).

23. Solve: $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$. Represent the solution set on the number line.

OR

Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

24. Evaluate**:** $\sin 10^\circ . \sin 50^\circ . \sin 70^\circ$.

25. How many five letter words with or without meaning each of 3 vowels and 2 consonants can be formed from the letters of the word "EDUCATION"**.**

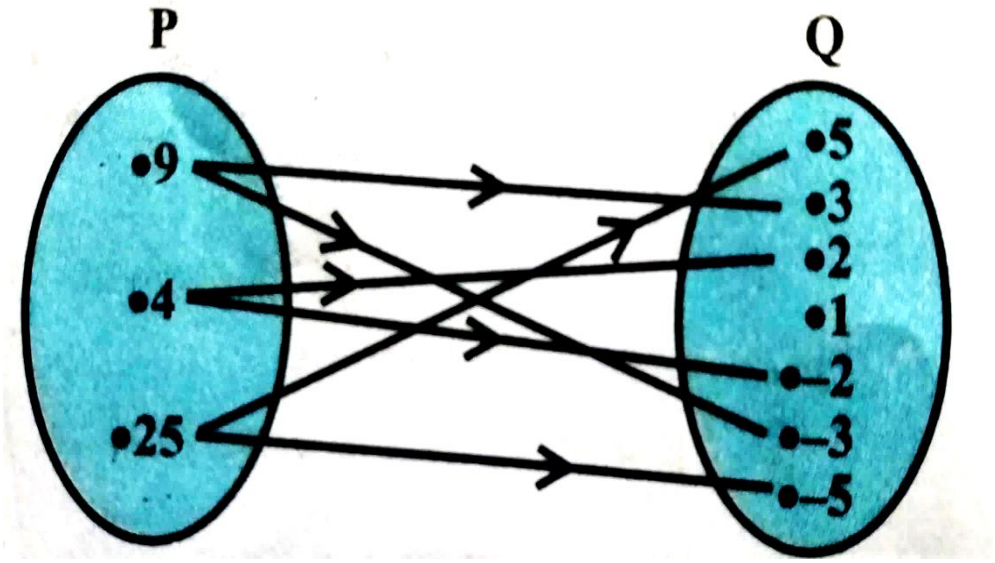
SECTION–C

This Section comprises short answer (SA) type questions of 3 marks each.

26. If C and D are two sets where $C \cap X = D \cap X = \phi$ and $C \cup X = D \cup X$ for some set X, Prove that C = D.

27. For the given sets A, B and C, prove that $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = (B \cap C')$.

28. The following figure shows a relation between the sets P and Q. Write this relation (i) in set builder form (ii) in roster form .What is its domain and range.



OR

Draw the graph of $f(x)$, if the function f is defined by $f(x) = \begin{cases} 1-x, x<0 \\ 1, \quad x=0 \\ x+1, x>0 \end{cases}$

Find the domain and range of f.

29. The sum of the coefficients of the first three terms in the expansion of $(x - \frac{3}{x^2})^m$, $x \neq 0$, m being a natural number is 559. Find the term of the expansion containing x^3.

OR

Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.....(2n-1)}{n!}\, 2^n x^n$, where n is a positive integer.

30. Express the conjugate of the complex number $z = \frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ in $(a + ib)$ form.

OR

If $(a + ib) = \frac{(x+i)^2}{2x^2+1}$, prove that $(a^2 + b^2) = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

31. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then prove that $\tan\theta = \frac{1-m}{1+m}\cot\phi$.

SECTION–D

This section comprises long answer (LA) type questions of 5 marks each.

32. (i) If f $(x + 1) = x^2 - 3x + 2$, then find f (x).

(ii) Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a linear function on Z. Find $f(x)$.

33. Prove that: $\sin 3x \sin^3 x + \cos 3x \cos^3 x = \cos^3 2x$.

OR

Find the value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$.

34. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, Find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

35. Find the number of words with or without meaning which can be made using all the letters of the word "AGAIN". If these words are written as in a dictionary, what will be the 50^{th} word?

OR

A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of

(i) exactly 3 girls ? (ii) at least 3 girls ? (iii) at most 3 girls ?

SECTION–E

This section comprises 3 case study based questions of 4 marks each.

Case Study−1

36.

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An organization conducted bike race under 2 different categories-boys and girls. There were 250 participants in total. Among them, three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = $\{b_1, b_2, b_3\}$, G = $\{g_1, g_2\}$, where B represents the set of boys and G be the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions.

Based on the above information answer the following questions:

(i) Write (G × B). **1**

(ii) Ravi wishes to form all the relations possible from B to G. How many such relations are possible? **1**

(iii) Let R is a relation on B defined by R = $\{(x, y)$: height of x is 2cm more than that of y$\}$, For all x, y, z ∈B, if $(x, y) \in R$ and $(y, z) \in R$, then whether $(x, z) \in R$? Justify. **2**

OR

Find the number of non-empty relations possible from B to B. **2**

Case Study −2

37. Abhipsa was playing in a park, which was having the shape of a right angled triangle. So she was trying to correlate her learning skills with the right triangular park. During her study of triangles relating to the angles with the ratios of their sides, she imagined

that the two different angles A & B may or may not be of the same triangle but she obtained the value of cosA = $\frac{3}{5}$ and cosB = $-\frac{12}{13}$, where $0 < A < \frac{\pi}{2}$, $\pi < B < \frac{3\pi}{2}$

Based on the above information answer the following questions:

(i) Find the value of $(\sin A + \sin B)$. **1**

(ii) Find the value of sin 2A. **1**

(iii) Find the value of $\tan(A - B)$. **2**

OR

Find the value of $\cot(A + B)$. **2**

Case Study−3

38. In a company there are 8 men and 9 women officers. The manager of the company wants to form a core committee of 12 members to take necessary decisions. The manager also has to take care of different aspects for the benefit of the company.

Based on the above information answer the following questions:

(i) Find the number of ways in which at least 7 men included in the committee. **2**

(ii) Keeping the idea of women empowerment in mind, if the manager wants to take the majority of female members in the committee, then find the number of ways to form the committee. **2**