PRACTICE PAPER-5

General Instructions:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections-**I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections-**III, IV and V**.
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks each**.
5. Internal choice is provided in **3** questions of Section-III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part – A

Section – I

All questions are compulsory. In case of internal choices attempt any one.

1. If E_1 is an equivalence class with respect to relation R defined on set A and $i = 1, 2,.., 5$. If $x \in E_i$ and $y \in E_j$, $i \neq j$, then can we say x and y are related to each other with respect to relation R?

OR

Show that the function $f : N \to N$, given by $f(x) = 2x$ is not onto.

2. If R is an equivalence relation defined in set

$A = \{1, 2, 3, .., 10\}$ as $R = \{(a, b) : |a - b|$ is a multiple of $3\}$. Write the equivalence class $\{1\}$.

3. Show that the function $f(x) = \sin x, x \in [0, \pi]$ is not one-one.

OR

An identity relation is a reflexive relation but a reflexive relation may or may not be identity relation. Support your answer with an example.

4. If A is any non-zero matrix of order n, then find $A \cdot \text{Adj}\, A$, in terms of $|A|$ and identity matrix I.

5. For what value of $k \in N$, $\begin{vmatrix} k & 3 \\ 4 & k \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 0 & 1 \end{vmatrix}$?

OR

If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = p$, then find the value of $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$, if $p = 20$.

6. For the matrixes A and B, if multiplication is defined and $AB = A$ and $BA = B$, then find B^2.

7. Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

OR

Evaluate $\int \sin 2x \sin 3x \, dx$.

8. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

9. How many arbitrary constants are there in general solution of a different equation of order 1?

OR

Find general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$.

10. If vectors $2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other, then find the value of λ.

11. Prove that $|\vec{a} \times \vec{b}|^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$.

12. Find the position vector of a point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio 1 : 2 internally

13. If A and B are two events such that $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\overline{B}) = \frac{1}{2}$. Then find whether the events A and B are independent or not.

14. The probability of solving a problem by A and B is $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Then find the probability that problem will be solved.

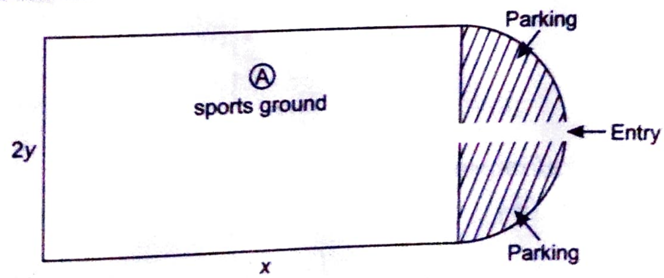
15. Find direction cosines of normal to the plane $2x - 3y + 6z = 7$.

16. Find the direction ratios of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Section – II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. The government of a state, which has mostly hilly area decided to have adventurous playground on the top of hill having plane area and space for 10000 persons to sit at a time. After survey it was decided to have rectangular play ground with a semicircular parking at one end of play ground only as space is less. The total perimeter of the field is measured as 1000 m as shown



Based on the above information answer the following.

(*i*) Looking at the figure (plan) the relation between x and y is

(*a*) $x + 2y + \pi y = 1000$

(*b*) $x + 2y + \pi y = 500$

(*c*) $2x + 2y + \pi y = 1000$

(*d*) $x + y + \pi y = 1000$

(ii) Area of sports ground in terms of x is

(a) $\frac{2}{\pi+2}(1000x - 2x^2)\text{m}^2$ (b) $\frac{1}{\pi}(1000x - 2x^2)\text{m}^2$

(c) $\frac{2}{\pi+2}(500x - 2x^2)\text{m}^2$ (d) $\frac{1}{\pi}(500x - 2x^2)\text{m}^2$

(iii) The maximum area of sports ground is for x equal to

(a) 500 m (b) 50 m

(c) 100 m (d) 250 m

(iv) The government wants to maximise the area including parking area for this to happen, value of y is

(a) $\frac{1000}{4+\pi}$ m (b) $\frac{2000}{4+\pi}$ m (c) $\frac{500}{4+\pi}$ m (d) $\frac{750}{4+\pi}$ m

(v) What is the maximum area of the sports field alone?

(a) $\frac{90000}{2+\pi}\text{m}^2$ (b) $\frac{160000}{2+\pi}\text{m}^2$ (c) $\frac{250000}{2+\pi}\text{m}^2$ (d) $\frac{100000}{2+\pi}\text{m}^2$

18. During the time of need and otherwise also people help the needy. It was found in survey that out of 200 people surveyed in a city 50 help the needy on regular basis, 120 contribute to prime Minister relief fund and the rest help through NGO's. A person is selected who is in the need of a help, the probabilities of help through persons on regular basis from Prime Minister relief fund and through NGO's are 0.15, 0.06 and 0.10 respectively.

Based on the above information answer the following.

(i) The probability of a help through NGO's is

(a) $\frac{1}{4}$ (b) $\frac{17}{20}$ (c) $\frac{5}{12}$ (d) $\frac{3}{20}$

(ii) The conditional probability of helping the needy through Prime Minister's relief fund is

(a) 0.15 (b) 0.06 (c) 0.10 (d) 0.69

(iii) The probability that the needy person received the help is

(a) 0.177 (b) 1.77 (c) 0.0885 (d) 0.67

(iv) The probability that needy person was helped through person on regular basis is

(a) $\frac{75}{177}$ (b) $\frac{72}{177}$ (c) $\frac{1}{59}$ (d) $\frac{6}{59}$

(v) In a city of population 1 lac how many are expected to help on regular basis?

(a) 25000 (b) 60000 (c) 20000 (d) 5000

Part – B

Section – III

All questions are compulsory. In case of internal choices attempt any one.

19. Find the value of, $\tan^{-1}(1) + \tan^{-1}(-\sqrt{3})$.

20. Find a matrix X such that $2A + B + X = O$, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$.

OR

If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then find the value of k.

21. Find the value of k so that the function f defined by $f(x) = \begin{cases} kx+1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$.

22. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

23. Evaluate $\int \frac{3ax}{b^2 + c^2x^2} dx$

OR

Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left|\frac{2 - \sin x}{2 + \sin x}\right| dx$

24. Find the area bounded by the curve $y = \cos x, x \in [0, \pi]$.

25. Find equation of the curve passing through (1, 1) and satisfying the differential equation $\frac{dy}{dx} = \frac{2y}{x}$.

26. Two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$, represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of the median through A.

27. Write the Cartesian equation of the following line given in vector form: $\vec{r} = 2\hat{i} + \hat{j} - 4\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$.

28. A and B throw a pair of dice turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is $\frac{9}{17}$.

OR

A random variable X, has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	$2p$	$2p$	$3p$	p^2	$2p^2$	$7p^2$	$2p$

Find the value of p.

Section – IV

All questions are compulsory. In case of internal choices attempt any one.

29. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5; T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

30. If $y = 3at^2; x = 5bt^4$, find $\frac{d^2y}{dx^2}$ at $t = 1$.

31. Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable for $x \in R$.

OR

Find $\frac{dy}{dx}$, if $y = (\log x)^x + x^{\log x}$.

32. Show that equation of the tangent to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_0, y_0) is $\frac{x x_0}{a^2} + \frac{y y_0}{b^2} = 1$.

33. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\cot x}}$

34. Find the area of the region bounded by the lines $y = 4x + 5, x + y = 5$ and $x - 4y + 5 = 0$.

OR

Make a rough sketch of the region given below and find its area using integration $\{(x, y): 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$.

35. Find the general solution of the differential equation $y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$.

Section – V

All questions are compulsory. In case of internal choices attempt any one.

36. If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. How we can use A^{-1} to solve the system of equations $\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 2; \frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 5;$ $\frac{6}{x}+\frac{9}{y}-\frac{20}{z} = -4.$

OR

If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence show that how we can use A^{-1} to solve the system of equations $2x + y - 3z = 13;$ $3x + 2y + z = 4, x + 2y - z = 8.$

37. Find the vector and Cartesian equations of the plane containing the two lines $\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$.

OR

Find the equation of the plane passing through the point (3, 2, 0) and contains the line $\frac{x-3}{1} = \frac{y-6}{5}; z = 2.$

38. Solve the following linear programming problem (LPP) graphically?

Maximise $Z = 20x + 40y$

subject to constraints

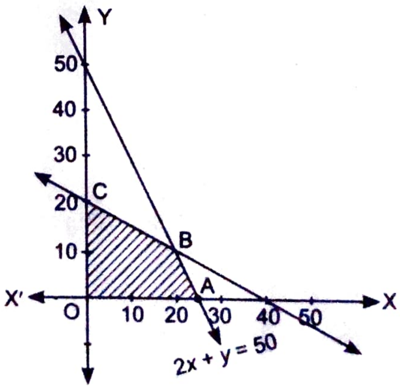
$1.5x + 3y \le 42 \Rightarrow 15x + 30y \le 420$

$\Rightarrow x + 2y \le 28$

and $3x + y \le 24, x \ge 0, y \ge 0.$

OR

Taking variables x and y along x-axis and y-axis in the following graph, shaded portion represents the feasible solution of *LPP*.



Answer each of the following:

(i) Form the constraints with respect to given feasible region for *LPP*.

(ii) If $Z = x + y$ is objective function, find maximum value of Z.