

MATHEMATICS

(041)

LEARNING OUTCOMES :

The broad objectives of teaching Mathematics at senior stage intend to help the students:

- To acquire knowledge and critical understanding, particularly by way of visualization, of basic concepts, terms, principles, symbols and mastery of underlying processes and skills.
- To meet the flow of reasons while proving a result or solving a problem.
- To apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method.
- To develop positive attitude to think analyze and articulate logically.
- To develop interest in the subject by participating in related competitions.
- To acquaint students with different aspects of Mathematics as a discipline.
- To develop awareness of the need for national integration, protection of environment, observations of small family norms, removal of social barriers, elimination of gender biases.
- To develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics

ASSIGNMENT

Ch.1 SETS

1. If a set have n no. of elements then how many no. of subsets and how many no. of proper subsets?
2. Define equivalent sets and equal sets with examples.
3. If A, B and C are sets, prove that A subset of B and B subset of C implies A subset of C .
4. Write down all the subsets of set $A = \{-1, 0, 2\}$ hence write its power set.
5. List all the proper subsets of $\{0, 1, 2, 3\}$
6. State and prove De Morgans Laws.
7. Show that $A \cap B$ and $A \cap C$ need not imply $B = C$ with example.
8. In a class of 50 students , 30 students like Mathematics, 25 like Science and 16 like both. Find the number of students who like
1) Either Mathematics or Science (39)
2) Neither Mathematics nor Science.(11)
9. A and B are two sets such that $n(A-B) = 14+x, n(B-A)=x$. Draw a Venn diagram to illustrate this information. If $n(A)=n(B)$, find (1) the value of x (11) $n(A \cup B)$.
10. For any sets A and B , prove that $P(A \cap B) = P(A) \cap P(B)$.
11. In a survey it was found that 21 people liked product A , 26 liked product B and 29 liked product C . If 14 people liked product A and B , 12 people liked product A and C , 14 liked product B and C and 8 liked all the three products, find how many liked product C only.
12. In a survey of 600 students in a school, 150 were taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee.

Ch. 2 Relations and Functions

1. If the ordered pair $(x-1, y+3)$ and $(2, x+4)$ are equal, find x and y .
2. If $A=\{1,2\}$, $B=\{2,3\}$ and $C=\{0\}$ find $A*B*C$.
3. Find a and b if:
 - i. $(a+1, b-2) = (3, 1)$.
 - ii. $(\frac{a}{3} + 1, b - \frac{2}{3}) = (\frac{5}{3}, \frac{1}{3})$
 - iii. $(2a, a + b) = (6, 2)$
 - iv. $(a + b, 3b - 2) = (7, -2)$
4. If $P = \{1, 2\}$, form the set $P \times P \times P$.
5. Given that $B = \{2, 3, 5\}$ and some elements of $A \times B$ are $(a, 2), (b, 3), (c, 5)$. Find the Set A and the remaining ordered pairs of $A \times B$ such that $A \times B$ is the least.
6. If a relation $R = \{(0, 0), (2, 4), (-1, -2), (3, 6), (1, 2)\}$, then
 - i. Write domain of R .
 - ii. Write range of R .
 - iii. Write R in the set builder form.
 - iv. Represent R in arrow diagram.
7. $A = \{-1, 2, 5, 8\}$, $B = \{0, 1, 3, 6, 7\}$ and R be the relation "is one less than" from A to B , then a) find R as a set of ordered pairs.
b) find domain and range of R .
8. Let R be the relation on N defined by : $R = \{(a, b) : a \in N, b \in N \text{ and } a + 3b = 12\}$, then
 - a) list the elements of R
 - b) find the domain of R
 - c) find the range of R .
9. Which of the following are functions ? Give reasons. If it is a function, determine its domain and range.
 - i. $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$
 - ii. $\{(a, b), (b, c), (c, d), (d, e)\}$
 - iii. $\{(1, 2), (3, 1), (1, 3), (4, 1)\}$
 - iv. $\{(2, 1), (0, -1), (3, 1), (5, 4), (-1, 0), (3, 4), (1, 0)\}$
10. Let 'f' be a function defined by $f : R \rightarrow R$ as $f(x) = 4x + 7$, when $x \leq 3$ and $f(x) = 4$, when $x > 3$. Show that f is a function.
11. If f is a real function defined by $f(x) = x + \frac{1}{x}$, $x \in R - \{0\}$,
then show that $(f(x))^3 = f(x^3) + 3f(\frac{1}{x})$.
12. Find the domain and range of the following functions:
 - a) $f(x) = \frac{1}{x-2}$ b) $f(x) = \frac{x}{x+5}$ c) $f(x) = \frac{x}{x^2+3}$ d) $f(x) = \frac{1}{(2x-3)(x+1)}$ e) $f(x) = \frac{1}{\sqrt{9-x^2}}$,
13. find the range of the following functions :
 - a) $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in R \right\}$
 - b) $f = \left\{ \left(x, \frac{1}{1-x^2} \right) : x \in R, x \neq \pm 1 \right\}$.
14. Find the domain of the following real functions:
 - a) $f(x) = \sqrt{7-x} + \frac{3}{\sqrt{x^2-4}} + 9$ Ans. $(-\infty, -2) \cup (2, 7)$

$$b) f(x) = \sqrt{\frac{3-|x|}{4-|x|}}$$

$$\text{Ans. } [-3,3] \cup [-\infty, -4) \cup (4, \infty).$$

15. Draw graph for the following functions :

a) $f(x) = |x - 2| + |2 + x|, -3 \leq x \leq 3. \in$

b) $f(x) = 3 - x^2 \quad x \in R.$

c) $f(x) = \sqrt{4 - x^2} \quad x \in R.$

d) $f(x) = |x|.$

e) $f(x) = [x]$ for greatest integer function.

f) $f(x) = x^3 - 3.$

Ch.3 TOPIC-TRIGONOMETRY

1 A wire 121 cm long is bent so as to lie along the arc of a circle of 180 cm radius .Find degrees the angle subtended at the centre of the arc .

2 Find the angle between the hour hand and minute hand when the is 7:20 a.m.

3 If $\tan x + \cot x = 2$ find $\tan^4 x + \cot^4 x$

4 Prove the following:

i) $\cos 20^\circ \cos 30^\circ \cos 40^\circ \cos 80^\circ = \frac{\sqrt{3}}{16}.$

ii) $\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}.$

5. If $\tan x = p + 1$ and $\tan y = p - 1$, show that $2 \cot(x - y) = p^2$

6. Prove that: $\cos 2x \cos 2y + \cos 2(x + y)^2 - \cos(x - y) = \cos(2x - 2y)$

7. If $\tan \beta = \frac{n \sin \alpha \cos \alpha}{1 - n \sin^2 \alpha}$, show that $\tan(\alpha - \beta) = (1 - n) \tan \alpha.$

8. Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

9. Prove that $\frac{\sin(x+y) - 2 \sin x + \sin(x-y)}{\cos(x+y) - 2 \cos x + \cos(x-y)} = \tan x$

10. Prove that $\frac{\sec 8x - 1}{\sec 4x - 1} = \frac{\tan 4x}{\tan 2x}.$

Prove that: $\cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x$

11. Solve the following equations:

1. $3 \tan x + \cot x = 5 \operatorname{cosec} x.$

2. $\sqrt{3} \cos x - \sin x = 1.$

3. $\sin x + \sin 3x + \sin 5x = 0.$

4. $\tan x + \tan 2x + \sqrt{3} \tan x \tan 2x = \sqrt{3}.$

12. Show that $1 + \tan \theta \tan \frac{\theta}{2} = \tan \theta \cot \frac{\theta}{2} - 1 = \sec \theta .$

13. Show that : $\tan(x - y) + \tan(y - z) + \tan(z - x) = \tan(x - y) \tan(y - z) \tan(z - x).$

14. Show that $\frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A} = \cot 2A.$

15. If $\theta + \varphi = \alpha$ and $\tan \theta = k \tan \varphi$, then prove that : $\sin(\theta - \varphi) = \frac{k-1}{k+1} \sin \alpha .$

16. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then show that $\cot(A - B) = \frac{1}{x} + \frac{1}{y} .$

17. Show that : $\tan 75^\circ + \cot 75^\circ = 4.$

18. If $\tan \frac{x}{2} = \frac{m}{n}$, prove that $m \sin x + \cos x = n.$

19. Prove that: In any triangle ABC, $(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2} = c^2$

20. In any triangle ABC prove that

1. $\frac{b^2-c^2}{a^2} \sin 2A + \frac{c^2-a^2}{b^2} \sin 2B + \frac{a^2-b^2}{c^2} \sin 2C = 0$

2. $(b^2-c^2) \cot A + (c^2-a^2) \cot B + (a^2-b^2) \cot C = 0.$

21. Show that $\sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = 2 \cos \theta.$

22. If $\tan x = \frac{-4}{3}$ and $\frac{\pi}{2} < x < \pi$, find the value of $\sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}.$

23. If $A + B + C = \pi$, prove that

i. $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

ii. $\sin 2A - \sin 2B + \sin 2C = 4 \cos A \sin B \cos C$

iii. $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

iv. $\cos 4A + \cos 4B + \cos 4C = -1 + 4 \cos 2A \cos 2B \cos 2C$

v. $\sin A + \sin B - \sin C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$

vi. $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$

vii. $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2(1 + \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}).$

viii. $\tan A + \tan B + \tan C = \tan A \tan B \tan C.$

ix. $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

x. $\cot B \cot C + \cot C \cot A + \cot A \cot B = 1.$

24. In a triangle ABC, if $a \cos A = b \cos B$, show that the triangle is isosceles or right triangle.

Ch.4 Principle of Mathematical Induction.(not to be assessed in the final examination)

1) If P(n) is the statement " $n^3 + n$ is divisible by 3", show that P(3) is true but P(4) is not true.

2) Prove by mathematical induction that sum of first n odd natural numbers is equal to n^2

3) Prove by induction that $1^3 + 2^3 + 3^3 + \dots + n^3 = (\frac{n(n+1)}{2})^2.$

4) By induction, prove that $41^n - 14^n$ is divisible by 27 for all n belongs to N.

5) Prove that

$5^n - 5$ is divisible by 4 for all $n \in$

$N.$ Hence prove that $2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24 for all $n \in N.$

6) Use induction to prove that $n(n+1)(n+2)$ is divisible by 6 for all $n \in N.$

7) Using PMI prove that : $7+77+777+\dots$ up to n terms $= \frac{7}{81} (10^{n+1} - 9n - 10),$ for all $n \in N$

8) Using PMI prove that $x^{2n} - y^{2n}$ is divisible by $x - y$ for all $n \in N$

9) Using PMI prove that : If n is any odd positive integer then prove that $n(n^2 - 1)$ is divisible by 24.

10) Using PMI prove that $\frac{1}{3.6} + \frac{1}{6.9} + \frac{1}{9.12} + \dots + \frac{1}{3n(3n+3)} = \frac{n}{9(n+1)}$

Ch.5 Complex number and Quadratic equations

A number of the form $a + i b$, a and b are real numbers is called a complex number.

Where i (iota) is the imaginary number such that the iota square is equal to -1.

In $z = a + ib$ real z i.e. $\text{Re}(z)$ is a and imaginary i.e. $\text{Im}(z)$ is b

If $z_1 = a_1 + b_1i$ and $z_2 = a_2 + b_2i$ be two complex numbers then

$$z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$$

$$z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$$

$$|z| = \sqrt{x^2 + y^2} \text{ (Modulus of } z \text{)}$$

1. Express each of the following in the form $a + ib$:

i. $(1-i)^2$

ii. $(-2 - \frac{1}{3}i)^3$

iii. $(2i - i^2)^2 + (1 - 3i)^3$

iv. $(i^{18} + (\frac{1}{i})^{25})$

v. $i^n + i^{n+1} + i^{n+2} + i^{n+3}, n \in N$

2. Show that the real part of $(2+3i)(2-3i)(1+i)$ is zero.

3. If $a = 3+2i$ and $b = 3-2i$, then show that : $a^2 + ab + b^2 = 23$

4. If $(\cos \theta + i \sin \theta)^2 = x + iy$, then show that $x^2 + y^2 = 1$.

5. Find the real values of x & y for the following functions :

i. $2x + iy = 4 + 5i$

ii. $\frac{x-i}{3+2i} + \frac{x+i}{3-2i} = \frac{1+3i}{1-3i}$

iii. $(3y - 2)i^{16} + (5 - 2x)i = 0$.

iv. $(\frac{1}{3}x - \frac{1}{4}y) + \frac{3}{4}yi = -3 + 5i$.

v. $3x + 2yi + 5 - 2i = 7 - 3i$.

6. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that : $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

7. Find the multiplicative inverse of the following:

i. $1 - i$

ii. $4 - \sqrt{-9}$.

iii. $(3+i)(1+2i)(1-i)$.

iv. $\frac{2-3i}{3+2i}$

8. Write the additive inverse of following :

i. $1 + 2i$.

ii. $-3+2i$.

iii. $-5i - 3$

iv. $7 - 2i$.

9. Write the modulus and argument, hence write in polar form of the following :

i. $-1 + i$

ii. $4\sqrt{3} + 4i$

iii. $\frac{1+2i}{1-(1-i)^2}$

iv. $\frac{(1+i)^{13}}{(1-i)^7}$

v. $\frac{1+2i}{1-3i}$

vi. $4 - 4\sqrt{3}$

10. Find the modulus of the following:

- i. $(3+4i)(4+i)$
- ii. $\frac{(1+i)(2+i)}{3+i}$
- iii. $\frac{(3+4i)(4+5i)}{(4+3i)(6+7i)}$
- iv. $(4+3i)(5\sqrt{3}+3i)$

11. If $(1+i)(1+2i)(1+3i)\dots\dots\dots(1+ni) = x + iy$, show that $2.5.10\dots\dots\dots(1+n^2) = x^2 + y^2$.

12. Find the square root of following: (Not to be assessed in the final examination)

- i. $7 - 24i$
- ii. $\frac{2+3i}{5-4i} + \frac{2-3i}{5+4i}$
- iii. $3+4\sqrt{7}i$
- iv. $1 - i$
- v. $-15 - 8i$
- vi. $5 - 12i$
- vii. $-15 + 8i$

13. Write the complex numbers in their polar form:

- i. $[2(\cos 210^\circ + i \sin 210^\circ)][4(\cos 120^\circ + i \sin 120^\circ)]$
- ii. $[3(\cos 225^\circ + i \sin 225^\circ)][6(\cos 45^\circ + i \sin 45^\circ)]$

Note : Quadratic Equation Not to be assessed in the final examination .

Ch.6 Linear Equations:

Two real numbers or two algebraic expressions connected by the sign $<$, $>$, \leq or \geq is called an inequality.

1. Solve the inequality $3-2x \geq x-32$, given that a) $x \in \mathbb{N}$ b) $x \in \mathbb{I}$
2. Solve the inequality $5x - 3 < 3x + 1$ when a) x is an integer and b) x is a real number.
3. Solve the following inequalities for real x :
 - i. $3(2-x) \geq 2(1-x)$
 - ii. $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$
 - iii. $\frac{2x+1}{3} \geq \frac{3x-2}{5}$
4. Solve the following system of linear inequalities :

$$2(2x+3) - 10 < 6(x-2), \frac{2x-3}{4} + 6 \geq 4 + \frac{4x}{3}$$
5. Solve the following inequalities , also represent the solutions on the number line.
 - i. $2x - 5 \leq 5x + 4 < 11$
 - ii. $2x - 3 < 5x + 3 < 2x + 3$
 - iii. $2x - 3 \geq x + \frac{1-x}{3} > \frac{2x}{5}$.
 - iv. $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$
6. Solve the following inequalities graphically :
 - i. $x - 2y \geq 0, 2x - y + 2 \leq 0, x \geq 0$
 - ii. $-3x + 2y \leq 3, 2x + 3y < 12, 2x + 3y \geq 6$
 - iii. $x + 2y \leq 0, 3x + 4y \geq 12, x \geq 0, y \geq 1$.

- iv. $x+y < 5, 4x + y \geq 4, x+5y \geq 5, x \leq 4, y \leq 3.$
- v. $x + y \leq 6, x + y > 4.$
- vi. $x+2y < 8, 2x+y \leq 8, x, y \geq 0.$
- vii. $3x + 2y \geq 24, 3x + y \leq 15, x \geq 4, y \geq 0.$
- viii. $2x + y \leq 24, x + y < 11, 2x + 5y \leq 40, x \geq 0, y \geq 0.$
- ix. $3y - 2x \leq 4, x + 3y > 3, x + y \geq 5, y < 4.$
- x. $3y - 2x < 4, x + 3y > 3$ and $x + y \leq 5.$

Ch. 7 Permutation and Combinations

Factorial notation : $1!, 2!, 5!, 7!, 1000!$

$n!$ means the product of first n natural numbers and $0! = 1$, and $1! = 1$

$${}^n P_r = \frac{n!}{(n-r)!} \text{ And } {}^n C_r = \frac{n!}{r!(n-r)!}$$

1. Evaluate $\frac{8!}{6!5!}$
 2. How many 4 digit numbers are there with no digit repeated ?
 3. In how many ways can the letters of the word "ALTERNATELY" be arranged if the words start with L and end with Y.
 4. If ${}^n C_{10} = {}^n C_6$ find ${}^n C_1$.
 5. Determine n if a) ${}^{2n} C_3 : {}^n C_2 = 12 : 1$, ${}^{2n} C_3 : {}^n C_3 = 11 : 1$.
 6. If ${}^n C_{r-1} = 36$, ${}^n C_r = 84$ and ${}^n C_{r+1} = 126$, find ${}^n C_2$
 7. A convex polygon has 44 diagonals. Find the no. of its sides.(11)
 8. You can go from Delhi to Agra either by car or by bus or by train or by air. In how many ways can you plan your journey from Delhi to Agra ? (4)
- How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated ? (504)
9. There are 15 points in a plane, no three of which are in the same straight line except 4 which are collinear. Find the number of :
 - i) No girl ii) at least 3 girls. iii) atleast one girl and one boy? (21,91,441)
 10. How many words can be made by using all letters of the word "MATHEMATICS" in which all the vowels are never together ?
 11. If all the letters of following words be arranged in a dictionary to make words with or without meaning find total no. of words and the RANK of each word :
 - i. PRIYA
 - ii. LALITA
 - iii. ROMITA
 - iv. KUSHULTA
 - v. ARUSHIKA
 - vi. DURYODHAN
 - vii. DUSHASHAN
 - viii. RAMLALA
 - ix. MEGHNATH
 - x. KUNTEYE

Ch.8 BINOMIAL THEOREM.(Not to be assessed in the final examination .)

General expansion $(a + b)^n = C(n,0)a^n + C(n,1)a^{n-1}b + C(n,2)a^{n-2}b^2 \dots \dots \dots C(n,n)b^n$

General term of expansion $(a + b)^n$ is $C(n,r)a^{n-r}b^r$.

1. Expand by using Binomial theorem
 a) $(2x + 3y)^4$. b) $\left(\frac{2}{3y} - \frac{3p}{2}\right)^3$ c) $(4x + 5z)^4$ d) $(98)^5$ e) $(199)^4$ f) $(1001)^4$.
 2. Simply $(x + y)^6 + (x - y)^6$ and hence evaluate $(\sqrt{3} - 1)^6 + (\sqrt{3} + 1)^6$.
 3. Write down middle term in the expansion $(1 + x)^{2n}$.
 4. Which is larger $(1.01)^{100000}$ or 10000.
 5. Write the 5th term from end in the $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$, $x \neq 0$.
 6. Find the term independent of x in the expansion of $\left(2x - \frac{3}{x}\right)^{10}$.
 7. How many no. of terms in the expansion of $(a^2 - 2ab + b^2)^{10}$?
 8. How many no. of terms in the expansion of $(a^3 + 3ab + 2c)^{15}$?
 9. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.
 10. In $\left(3x^2 - \frac{1}{x}\right)^{18}$, which term contains x^{12} ?
 11. Find the values of following expansions by using binomial theorem:
 - i. $(1002)^3$
 - ii. $(999)^4$
 - iii. $(97)^3$
 - iv. $(501)^2$
 - v. $(899)^3$
 12. If p is the real number and the middle term in the expansion of $\left(\frac{p}{2} + 2\right)^8$ is 1120, find p. (p=2)
 13. Find the value of r, if the coefficients of $(2r+4)^{\text{th}}$ and $(r-2)^{\text{th}}$ in the expansion of $(1+x)^{18}$ are equal. (r=6)
 14. If the coefficient of second, third and fourth terms in the expansion of $(1+2x)^{2n}$ are in A.P., show that $2n^2 - 9n + 7 = 0$.
 15. Find the sixth term of the expansion $(y^{\frac{1}{2}} + x^{\frac{1}{3}})^n$, if the binomial coefficient of third term from the end is 45.
 16. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$ (-19)
 17. Find the coefficients of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$
 18. Find n in the expansion $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$ if the ratio of 7th term from the beginning to the 7th from end is $\frac{1}{6}$. (n=6)
 19. Find the total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ (51)
- Note: This whole chapter has been deleted for examination for this session.

Ch.9 Sequence and Series

Topic : Arithmetic Progression, Geometric Progression and Special Series :

- Find the number of integers lying between 100 and 1000 that are 1) divisible by 7 2) not divisible by 7
(1287,771)
- In an A.P. the m^{th} term is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ then Find p^{th} term, mn^{th} term and sum of mn^{th} terms.
- The sum of first four terms of an A.P. is 56. The sum of last four terms is 112. If first term is 11. Then find the no. of terms.
(11)
- If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. prove that a^2, b^2, c^2 are also in A.P.
- If a, b, c are in A.P. prove that $a(\frac{1}{b} + \frac{1}{c}), b(\frac{1}{c} + \frac{1}{a}), c(\frac{1}{a} + \frac{1}{b})$ are in A.P.
- If a, b, c are in A.P. prove that $a(\frac{1}{b} + \frac{1}{c}), b(\frac{1}{c} + \frac{1}{a}), c(\frac{1}{a} + \frac{1}{b})$ are in A.P.
- Show that $(x^2 + x y + y^2), (z^2 + z x + x^2)$ and $(y^2 + y z + z^2)$ are consecutive terms of an A.P. if x, y, z are in A.P.
- The product of three numbers in A.P. is 224 and the largest number is 7 times the smaller. Find the numbers.
(2,8,14)
- The digits of a 3-digit natural number are in A.P. and their sum is 15. The number obtained by reversing the digits is 396 less than the original number. Find the number.
- Find n so that $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be the A.M. between a and b .
- The sum of three numbers in G.P. is $\frac{13}{12}$ and their product is -1 . Find the numbers.
- If $x, 2y, 3z$ are in A.P. where the distinct numbers x, y, z are in G.P. then find the common ratio of the G.P.
- The sum of 4 numbers in G.P. is 60 and the arithmetic mean of the first and last is 18. Find the numbers.
- Find the four numbers in G.P. such that the sum of the extreme numbers is 112 and the sum of the middle numbers is 48.
- If S be the sum, P be the product and R be the sum of the reciprocals of 3 consecutive terms of a G.P., then find $P^2 R^3 : S^3$.
- If $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P. are themselves in G.P. show that p, q, r are in A.P.
- If a, b, c are in A.P. and x, y, z are in G.P. then show that
$$x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1.$$
- The lengths of three unequal edges of a rectangular solid block are in G.P. The volume of the block is 216 cm^3 and its surface area is 252 cm^2 . Find the length of longest edge.
(12)
- Find the sum of first 5 terms and first n terms of the sequence 3,6,12,.....
- If the sum of an infinite geometric series is 15 and sum of the squares of these terms is 45. Find the series.
- Find the sum of the series : $(x + y) + (x^2 + x y + y^2) + (x^3 + x^2 y + x y^2 + y^3) + \dots$ up to n terms.
- Find the sum of first n terms of the series : 1) $5 + 55 + 555 + \dots$ 11) $6 + 66 + 666 + \dots$ 111) $.7 + .77 + .777 + \dots$
- Find the sum of the series $1.2.3 + 2.3.4 + 3.4.5 + \dots$ Up to n terms
- $\frac{1.2^2 + 2.3^2 + \dots + n.(n+1)^2}{1^2.2 + 2^2.3 + \dots + n^2.(n+1)} = \frac{3n+5}{3n+1}$
- Find sum to n -terms of the series : $2+6+18+24+72+\dots$

Ch.10 Straight Lines

- Find the equation of a line passing through (2,3) parallel to the line $2x + 5y = 10$
- Find the area of the triangle whose vertices are (10,-6), (2,5) and (-1,3).
- If the vertices of a triangle are (1,k), (4,-3) and (-9,7) and its area is 15 sq. units. Find the value(s) of k.
- For what values of x are the points (1,5), (x,1) and (4,11) are collinear.
- If A(1,4), B(2,-3) and (-1,-2) are the vertices of a triangle ABC. Find
 - the equation of the median through A.
 - the equation of altitude through A.
 - the equation of right bisector of the side BC.
- Find the equation of a straight line which makes intercepts 3 and -5 on the coordinate axes.
- Find the equation of a line on which length of perpendicular from origin is 4 units and the line makes an angle of 120° with the positive direction of x-axis.
- A straight line passing through the point A(-1,2) has inclination $\frac{\pi}{3}$ and intersects the line $x + y = 5$ at P, find AP.
- If the slope of a line passing through the point A(3,2) is $\frac{3}{4}$, then find the point on the line which are 5 units away from the point A.
- In what direction should be a line be drawn through the point A(1,2) so that point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.
- Find the angle between the lines $y = (2 - \sqrt{3})(x + 5)$ and $(2 + \sqrt{3})(x - 7)$.
- Find the distance of the line $4x + 7y + 5 = 0$ from the point P(1,2) along the line $2x - y = 0$.
- If the image of the point (3,8) in the line $px + 3y - 7 = 0$ is the point (-1,-4), then find the value of p.
- Show that the points (1,1) and (2,-1) lie on the same side of the line $2x + 3y + 4 = 0$.
- In what ratio, the line joining (-1,1) and (5,7) is divided by the line $x + y = 4$
- If p is the length of perpendicular from origin to the line which makes intercepts a, b on the axes, prove that : $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$
- Find the image of the point P(-8,12) with respect to the line mirror $4x + 7y + 13 = 0$
- Find the points on the line $x + y - 3 = 0$ that are at a distance of $\sqrt{5}$ units from the line $x + 2y + 2 = 0$. (1,-4), (-9,6)
- A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the points (1,5). Obtain its equation.
- Find the equation of the straight line passing through the point of intersection of the lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ and perpendicular to the line $3x - 5y + 11 = 0$.

Ch.11 Conics Sections

- Find the equation of circle whose coordinates of the centre are (-2, 3) and radius is 4.
- Find the equation of a circle having (1,-2) as its centre and passing through the intersection of the lines $3x + y = 14$ and $2x + 5y = 18$.
- Find the equation of the circle whose centre is C(-2,3) and which touches the line $x - y + 7 = 0$.
- Find the equation of the circle which pass through the point (3,6) and touches both the axes.
- Find the equation of the circle when end points of the diameter are (-2,3) and (3,-5).
- Find the equation of the circle passing through the points (1,-2), (5,4) and (10,5).
- Show that the points (9,1), (7,9), (-2,12) and (6,10) are concyclic.

8. Find the equation of circle which passes through the origin and cut off chords of length s 4 and 6 on the positive side of the x-axis and y-axis respectively.
9. Find the equation of circle circumscribing the triangle formed by the lines:
a) $X + y = 6$, $x - y = 4$, $2x - y = 3$.
b) $x + y = 6$, $2x + y = 4$, $x + 2y = 5$.
10. Find the equation of parabola with vertex (0,0) and focus at (0,2).
11. AB is a line segment moving between the axes such that A lies on x-axis and B on y-axis. If P is a point on AB such that PA = b and PB = a. Find the equation of the locus of the point of P.
12. Find the axis, vertex, focus, length of latus-rectum and the equation of directrix for the following parabolas: (a) $y^2 = 12x$. (b) $y^2 = -8x$. (c) $x^2 = 16y$. (d) $x^2 = -9y$.
13. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus-rectum.
(18)
14. Find the equation of the parabola with a) focus at (-3,0) and directrix $x - 3 = 0$. b) focus at (0,4) and directrix $y + 4 = 0$: Also find the length of latus-rectum in each case.
15. Find the vertex, axis, focus, directrix, latus-rectum of the following parabolas
a) $x^2 - 8y - x + 19 = 0$
b) $4y^2 + 12x - 20y + 67 = 0$
c) $y = x^2 - 2x + 3$
d) $x^2 + 2y - 3x + 5 = 0$
16. Find the equation of parabola whose focus is (1,-1) and whose vertex is (2,1). Also find its axis and latus-rectum.
17. Find the equation of the parabola whose latus-rectum is 4 units, axis is the line $3x + 4y - 4 = 0$ and the tangent at vertex is the line $4x - 3y + 7 = 0$.
18. Find the coordinates of foci, vertices, the length of major and minor axis, latus-rectum and the eccentricity of the conic represented by the equation
a) $4x^2 + 9y^2 = 36$ b) $4x^2 + y^2 = 100$
19. Find the equation of the ellipse whose axis are parallel to the coordinate axes having its centre at the point (2,-3) and one focus at (3,-3) and vertex at (4,-3).
20. Find the equation of ellipse whose foci are (4,0) and (-4,0), eccentricity = 1/3.
21. Find the equation of the ellipse whose eccentricity is 2/3, length of latus-rectum is 5, centre is at origin and the major axis lies on x-axis.
22. Find the equation of ellipse with centre at origin, major axis along the x-axis, length of latus-rectum is 10 and the length of minor axis is equal to distance between foci.
23. If the latus-rectum of an ellipse is equal half the minor axis, then find its eccentricity.
24. If the eccentricity of an ellipse is 5/8 and the distance between its foci is 10, then find the latus-rectum of the ellipse.
25. Find the equation of the set of all points the sum of whose distances from the points (3,0) and (9,0) is 12.
26. Find the coordinates of foci, vertices, the length of major and minor axis, latus-rectum and the eccentricity of the conic represented by the equation
a) $5y^2 - 9x^2 = 36$ b) $y^2 - 16x^2 = 16$ c) $\frac{x^2}{9} - \frac{y^2}{36} = 1$ d) $9x^2 - y^2 = 225$

27. Find the equation of hyperbola, referred to its principal axes of coordinates, Vertex $(\pm 5, 0)$, Foci at $(\pm 7, 0)$ and Vertex at $(0, \pm 7)$, $e = \frac{4}{3}$.

Ch.12 Three Dimensional geometry

- Find the length of perpendicular drawn from the point $P(3, 4, 5)$ on y -axis.
- Find the equation of the locus of the point P , the sum of whose distances from the points $A(4, 0, 0)$ and $B(-4, 0, 0)$ is equal to 10.
- Find the point in XY -plane which is equidistant from three points $(2, 0, 3)$, $(0, 3, 2)$ and $(0, 0, 1)$.
Ans. $(3, 2, 0)$
- Find the coordinates of the centroid of the triangle whose vertices are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, and $C(x_3, y_3, z_3)$.
- Three consecutive vertices of a parallelogram $ABCD$ are $A(3, -1, 2)$, $B(1, 2, -4)$ and $C(-1, 1, 2)$. Find the fourth vertex D .
Ans. $(1, -2, 8)$
- Two vertices of a parallelogram are $(2, 5, -3)$, $(3, 7, -5)$ and its diagonal meet in $(4, 3, 3)$, find the remaining vertices of the parallelogram.
Ans. $(5, -1, 11)$
- Find the image of the point $(2, 0, -1)$ in the point $(3, 0, 4)$. Ans. $(4, -3, 9)$
- Find the Co-ordinates of the point equidistant from the points: $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ and $(0, 0, 0)$.
Ans. $(\frac{a}{2}, \frac{b}{2}, \frac{c}{2})$
- In what ratio is the line joining the points $A(-2, 3, 1)$ and $B(3, -2, 5)$ divided by ZX -plane?
Ans. $3 : 2$
- If A and B are the points $(-2, 2, 3)$ and $(-1, 4, -3)$ respectively, then find the locus of P such that $3|PA| = 2|PB|$.
- If $A(2, 2, -3)$, $B(5, 6, 9)$ and $C(2, 7, 9)$ are the vertices of the triangle ABC and the internal bisector of angle BAC meet at D , then find the coordinates of the point D .
- Find the coordinates of the centroid of the triangle, the mid points of whose vertices are $(1, 2, -3)$, $(3, 0, 2)$, and $(-1, 1, -4)$.

Ch. 13 Limits and Derivatives

- $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$
- $\lim_{x \rightarrow 0} \frac{(1+x)^5 - 1}{x}$
- $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$
- $\lim_{x \rightarrow \frac{1}{2}} \frac{8x^3 - 1}{16x^4 - 1}$
- $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$
- $\lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$
- $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3} = \frac{1}{16}$

8. $\lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\sin x - \cos x}{x - \frac{\pi}{4}} \right)$
9. Evaluate : $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\pi \sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}$
10. Find the derivative of following functions by using first principle method a) $x \sin x$ b) $\sin(2x+5)$ d) $\cos x^2$ e) $\sin \sqrt{x}$ f) $\tan \sqrt{x}$ g) $\cot(2x + 5)$
11. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, prove that $f'(\frac{\pi}{2}) = 1$.
12. If $y = (3x+5)(1+\tan x)$ find $\frac{dy}{dx}$.
13. If $y = \sqrt{\frac{x}{p}} + \sqrt{\frac{p}{x}}$, prove that $2xy \frac{dy}{dx} = \sqrt{\frac{x}{p}} + \sqrt{\frac{p}{x}}$.
14. Find the derivative of following functions w.r.t. x : a) $\sin x + x \cos x$, b) $\cos x^2$, c) $\sqrt{x \cos x + \sin x}$, d) $\frac{\sin x + \cos x}{\sin x - \cos x}$
15. If $y = x^n + x^{n-1} + x^{n-2} + \dots + 1$ find $\frac{dy}{dx}$.
16. If $y = e^x(x + \log x)$ find the $\frac{dy}{dx}$.
17. If $y = \frac{e^x + \sin x}{1 + \log x}$ find $\frac{dy}{dx}$.
18. If $f(x) = x^2 - 5x + 7$, find $f'(3)$ and $f'(\frac{7}{2})$ from definition. Hence show that $f'(\frac{7}{2}) = 2f'(3)$.
19. If $f(\theta) = \frac{1 - \tan \theta}{1 + \tan \theta}$ then find $f'(\frac{\pi}{4})$.
20. If $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots + \frac{1}{x^n}$ then find $f'(x)$.
21. If $y = \frac{x-1}{x+1} + \frac{x+1}{x-1}$ find $\frac{dy}{dx}$ at $x = 0$.

22. Using first principle find the derivative of $\sqrt{\tan 2x}$

23. Evaluate: (a) $\lim_{x \rightarrow 1/2} \frac{8x^3 - 1}{16x^4 - 1}$ (b) $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x\sqrt{a^2+ax}}$

Ch.14. Mathematical Reasoning. (Not to be assessed in the final examination)

The global of study of logic is to construct good or sound arguments, and to recognize bad or unsound arguments. Thus logic is the science of reasoning. This topic will include Propositions (Statements), Negations of statements, logical connectives and Quantifiers, converse contrapositive, Validating statements etc.

- Which of the following sentences are statements:
 - Mathematics is fun.
 - Today is a rainy day.
 - Answer this question.
 - There are always 31 days in a month.
 - Every parallelogram is a rectangle.

- f) Tomorrow is a holiday.
2. Write the negation of following:
 - i. Pakistan is a muslim country.
 - ii. India is a democratic country.
 - iii. Himachal is a beautiful place.
 3. Write the converse of the following statement:
If two lines are parallel, then they do not intersect in the same plane.
 4. What is the truth value of the statement: 100 is divisible by 3,5 and 11.
 5. Write the contrapositive of the statement: “ If a triangle is equilateral, it is isosceles “
 6. Write the component statements of the following compound statements :
 - a) 7 is both odd and prime number.
 - b) Jack and hill went up to the hill.
 - c) Ram is intelligent and sharp clever.
 7. Identify the type ‘or’ (inclusive or exclusive) used in the following :
 - a) Student can take French or Spanish as their third language.
 - b) $\sqrt{5}$ is rational or irrational number.
 - c) 175 is multiple of 5 or 8.
 8. Which of the following statements are true or false. Give reason.
 - a) 48 is multiple of 6,7 and 8.
 - b) Earth is flat or it revolves around the moon.
 9. Identify the Quantifier :
 - i. For every n , \sqrt{n} is a real number.
 - ii. There is a mathematician who is not a man.
 10. Prove the following statement by contradiction method : “The sum of an irrational number and a rational number is irrational”.
 11. Which of the following statements are true or false. Give reason.
 - i. Is a multiple of 6, 7 and 8.
 - ii. Earth is flat or it revolves around the moon.
 - iii. 78 is divisible by 3, 5 and 7.
 12. Verify by the contradiction that $\sqrt{11}$ is an irrational number.
 13. Write the converse of the statement : If a number n is even, then n^2 is even.
 14. Write the contra positive of following statement : If a triangle is equilateral, it is isosceles.
 15. Write the negation of the following statements :
 - i. All men are mortal.
 - ii. Is not a rational number.

Note : this chapter has been deleted for this session for examination.

Ch.15 Statistics

1. Find the mean deviation about the median for the following data :

| | | | | | | |
|-------|---|---|---|----|----|----|
| X_i | 5 | 7 | 9 | 10 | 12 | 15 |
| F_i | 8 | 6 | 2 | 2 | 2 | 6 |

2. Calculate the mean deviation about the mean for the following data :

| | | | | | | | | |
|----------------|-------|---------|---------|---------|---------|---------|---------|---------|
| Income per day | 0-100 | 100-200 | 200-300 | 300-400 | 400-500 | 500-600 | 600-700 | 700-800 |
| Number | 4 | 8 | 9 | 10 | 7 | 5 | 4 | 3 |

| | | | | | | | | |
|------------|--|--|--|--|--|--|--|--|
| of persons | | | | | | | | |
|------------|--|--|--|--|--|--|--|--|

3. Find the mean, variance and standard deviation for the following numbers:

25, 50, 45, 30, 70, 42, 36, 48, 34, 60.

4. Find the mean, S.D. and variance of a) first n odd natural numbers, b) first ten prime numbers.

5. Calculate the mean and standard deviation for the following data:

| | | | | | | | |
|--------------------------|-----|-----|-----|-----|-----|-----|-----|
| Wages per day upto(inRs) | 100 | 200 | 300 | 400 | 500 | 600 | 700 |
| Number of workers | 9 | 26 | 58 | 81 | 121 | 139 | 140 |

6. Prove that the standard deviation is independent of any change of origin, but is dependent of change of scale.

7. The mean and the standard deviation of 25 observations are 60 & 3 respectively. Later on it was decided to omit an observation which was incorrectly recorded as 50. Calculate the mean and standard deviation of remaining 24 observations.

8. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases.

i. If wrong item is omitted.

ii. If it is replaced by 12.

9. The mean and S.D. of marks obtained by 50 students of a class in three subjects Mathematics, Physics and Chemistry are given below :

| | | | |
|--------------------|-------------|---------|-----------|
| Subject | Mathematics | Physics | Chemistry |
| Mean | 42 | 32 | 40.9 |
| Standard Deviation | 12 | 15 | 20 |

Which of the three subjects shows the highest variability in marks and which shows the lowest?

10. The score of 48 children in an intelligence test are shown in the following frequency table:

| | | | | | | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|
| Score | 71 | 76 | 79 | 83 | 86 | 89 | 92 | 97 | 101 | 103 | 107 | 110 | 114 |
| Frequency | 4 | 3 | 4 | 5 | 6 | 5 | 4 | 4 | 3 | 3 | 3 | 2 | 2 |

Calculate the variance σ^2 and out the percentage of children whose score lie between $\bar{x} - \sigma$ and $\bar{x} + \sigma$.

11. Coefficient of variation of two distributions are 70 and 75, and their standard deviations are 28 and 27 respectively. What are their arithmetic means?

12. A panel of two judges graded seven dramatic performance by independently awarding marks as follows:

| | | | | | | | |
|-------------|----|----|----|----|----|----|----|
| Performance | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Marks by P | 46 | 42 | 44 | 40 | 43 | 41 | 45 |
| Marks by Q | 40 | 38 | 36 | 35 | 39 | 37 | 41 |

Find out coefficient of variation in the marks awarded by two judges and interpret the result.

Ch. 16 PROBABILITY

RANDOM EXPERIMENT: It is an experiment whose all possible outcomes are known, but it is not possible to predict the exact outcome in advance.

SAMPLE SPACE: The set of all possible outcomes of a random experiment is called the sample space denoted by S.

1. A coin is tossed twice write sample space. Also write sample space when coin tossed 3 and 4 times.
2. A die is thrown twice write sample space.
3. A coin is tossed and a die is thrown. Describe sample space.
 $P(A \cup B) = P(A) + P(B) + P(A \cap B)$.
4. A die is thrown, find the probability of following events:
 - i. A prime number will appear.
 - ii. A number greater than or equal to 3.
 - iii. A number less than one.
 - iv. A prime number less than 6.
 - v. An even prime number.
5. A fair coin with 1 marked on one face and 6 mark on other face and a fair die are both tossed find the probability that the sum of numbers 0turn up is a) 3, b) 12.
6. From a well shuffled deck of 52 cards two cards are drawn at random. Find the probability of drawing cards a) both cards spade. b) diamonds or aces. c) one king and other queen. d) both black cards.
7. There are three mutually exclusive and exhaustive events A,B and C. The odds are 8 : 3 against A and 2 : 5 in favour of B. Find odds against C. (43 :24)
8. In a given race, the odds in favour of four horses A,B,C and D are 1 : 3, 1 : 4, 1 :5 and 1 : 6 respectively. Assuming the dead heat is impossible, find the chance that one of them wins the race. (319:420)
9. Find the probability of at least two tails or at most two heads in tosses of three coins. (7:8)
10. From a well shuffled deck of 52 cards, a card is drawn at random. What is the chance that it a) is a queen or a jack. b) a king or a heart or a red card. (2/13, 7/13)
11. A pair of dice is rolled once. Find the probability of throwing a total of 6 or a multiple of 3 on only one die. (7/12)
12. A pair of dice is rolled twice in succession. Find the probability of obtaining a total of 5 or an odd number on each die. (13/36)
13. An integer is selected at random from amongst first one hundred positive integers. Find the probability that the integer chosen is either a multiple of 6 or a multiple of 8. (6/25)
14. Seven persons are to be seated in a row. Find the probability that two particular persons sit next to each other. (2/7)
15. Six boys and six girls have to sit in a row at random. Find the probability that
 - i)The six girls sit together
 - ii) the boys and girls sit alternately. (1/132, 1/462)
16. In a hand at whist, what is the chance that four kings are held by a specified player ?

17. Suppose that each child born is equally likely to be a boy or a girl. Consider a family with exactly three children

1V. Scheme of Sections :

Part A : 24 Objective Questions of 1 marks each.

Part B : Carries 56 marks each has Descriptive Type questions.

Both part A and B have choices

Part A: It contains two sections I and II.

Section I comprises 16 very short answers type questions.

Section II comprises 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part -B:

1. It consists of three sections – III, IV and V.
2. Section III comprises of 10 questions of two marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choices is provided in 3 questions of section III, 2 questions of section IV and 3 sections of section V.

Sample Paper-1

Section A

Q. 1 to 8 are multiple choice type questions. Select the correct option.

1. The modulus of $\left(\frac{1+i}{1-i} - \frac{1-i}{1+i}\right)$ is (1)

| | | | |
|------|------|------|------------------|
| a. 2 | b. 4 | c. 1 | d. None of these |
|------|------|------|------------------|

2. If $\cos x = \frac{4}{5}$ and x is an acute, then the value of $\tan 2x$ is (1)

| | | | |
|---------|---------|--------|--------|
| a. 7/24 | b. 24/7 | c. 3/4 | d. 4/5 |
|---------|---------|--------|--------|

3. The principal solution of $\sqrt{2} \cos x + 1 = 0$ is (1)

| | | | |
|-------------------------------------|-------------------------------------|-------------------------------------|------------------|
| a. $\frac{3\pi}{4}, \frac{4\pi}{3}$ | b. $\frac{3\pi}{4}, \frac{5\pi}{4}$ | c. $\frac{3\pi}{7}, \frac{7\pi}{4}$ | d. None of these |
|-------------------------------------|-------------------------------------|-------------------------------------|------------------|

4. If a Set have four elements then the number of proper subsets: (1)

| | | | |
|------|-------|-------|---------|
| a. 4 | b. 15 | c. 16 | d. zero |
|------|-------|-------|---------|

5. If AM of two positive numbers a and b is equal to their GM, then: (1)

| | | | |
|------------|------------|------------|----------------|
| a. $a < b$ | b. $a > b$ | c. $a = b$ | d. $a + b = 0$ |
|------------|------------|------------|----------------|

6. If $A = \{x; x \in N; x < 6\}$ and $B = \{x; x \in N; x > 8\}$ then $A \cap B$ is: (1)

| | | | |
|----------|-----------|--------|------------|
| a. {1,2} | b. ϕ | c. {7} | d. {1,3,5} |
|----------|-----------|--------|------------|

7. A relation R defined from {2,3,4,5} to {3,6,7,10} by R { (x,y): x and y are coprime }, Then the domain of R is : (1)

| | | | |
|------------|----------|------------|--------------|
| a. {2,3,5} | b. {3,5} | c. {2,3,4} | d. {2,3,4,5} |
|------------|----------|------------|--------------|

8. If A and B are two sets such that $A \cup B = A \cap B$ then: (1)

| | | | |
|------------------|---------------------|----------|------------------|
| a. A is Subset B | b. B is subset of A | c. A = B | d. None of these |
|------------------|---------------------|----------|------------------|

9. The value of $\frac{\tan^2 15^\circ - 1}{\tan^2 15^\circ + 1}$ is (1)

| | | | |
|--------------------------|-------------------------|------------------|-------------------|
| a. $\frac{-\sqrt{3}}{2}$ | b. $\frac{\sqrt{3}}{2}$ | c. $\frac{1}{2}$ | d. $\frac{-1}{2}$ |
|--------------------------|-------------------------|------------------|-------------------|

10. The number of terms in the expansion of $(2x + 3y - 4z)^6$ is (1)

| | | | |
|-------|-------|------|-------|
| a. 28 | b. 56 | c. 7 | d. 13 |
|-------|-------|------|-------|

Q.N 11-15 Fill in the blanks :

11. If $C_{(n,a)} = C_{(n,b)}$, then n is equal to (1)

OR

The value of $7! - 4!$ is

12. The derivative of $x \tan x$ is (1)

OR

The value of $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x - 2}$ is

13. The contrapositive of the statement : “ If you are born in India the you are citizen of India” is

14. The sum of first five terms of the sequence 3, 7, 11, 15..... (1)

15. The solution set for $25x < 100, \forall x \in N$ (1)

16. Find the value of $\tan 22^\circ + \tan 23^\circ + \tan 22^\circ \tan 23^\circ$ (1)

17. Find principle argument of the complex number $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$. (1)

OR

Write the conjugate of complex number $\frac{1}{1+i}$ in the form of $a + ib$.

18. Which term is the middle term in the expansion $(3a - 2b)^8$. (1)

19. Define Exhaustive events. (1)

20. A sum of Rs 6240 is paid off in 30 instalments, such that each instalment is Rs 10 more than the preceding instalment. Calculate the value of the first instalment. (1)

Section B

21. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is $(3,4)$ then find the coordinate of the other end of the diameter. (2)

OR

Find the equation of a parabola with the focus at $(-1, -2)$ and the directrix $x - 2y + 3 = 0$.

22. The centroid of a triangle ABC is at the $(1,1,1)$. If the co-ordinates of A and B are $(3, -5, 7)$ and $(-1, 7, -6)$ respectively. Find the co-ordinates of the point C. (2)

23. The probability of two events A and B are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither A nor B occurs. (2)

OR

In a single throw of two dice, find the probability of getting doublets.

24. If the arcs of the same length in the two circles subtend angles 65° and 110° at the center, find the ratio of their radii.

25. Find the real x and y if $\frac{x-2+(y-3)i}{1+i} = 1 - 3i$. (2)

26. In the expansion of $\left(x^2 + \frac{1}{x^2}\right)^{16}$, find the term independent of x. (2)

Section C

27. Out of 280 persons in a locality, 135 read a news paper A, 110 read news paper B, 80 read news paper C, 35 of these read news paper A and B, 30 read A and C, 20 read B and C. Also each person read at least one news paper of the three news papers. How many read all three news papers.

28. If $f(x) = x^3$, find $\frac{f(1.1) - f(1)}{1.1 - 1}$.

OR

Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z ? Justify your answer.

29. Using P.M.I for all n belongs to natural numbers, prove that :

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}.$$

30. Differentiate $\sqrt{\tan x}$ w.r.t. x by using first principle.

OR

Evaluate : $\lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$.

31. How many lawn-tennis mixed doubles games can be arranged from 7 married couples if no husband and wife pair is included in the game?

32. Solve the following system of equations graphically: Type equation here.

$$3y - 2x \leq 4, x + 3y \geq 3 \text{ and } x + y \leq 5.$$

Section D

33. Solve: $\tan x + \tan 2x + \sqrt{3} \tan x \tan 2x = \sqrt{3}$

OR

Prove that: $\cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x$

34. Find the distance of the line $4x + 7y + 5 = 0$ from the point $(1,2)$ along the line $2x - y = 0$.

35. find the mean, and variance for the following data :

| | | | | | |
|-------------|------|-------|-------|-------|-------|
| Classes | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
| Frequencies | 5 | 8 | 15 | 16 | 6 |

36. Find the probability that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards. it contains (i) all Kings (ii) 3 Kings (iii) at most two kings.

MODEL QUESTION PAPER II
SUB : MATHEMATICS

CLASS XI

Time : 3 hours Max Marks : 100

GENERAL INSTRUCTION

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted. You may ask for logarithmic tables if required.

Section A

Q.1 If $f(x) = x^2 - 3x + 1$ and $f(2\alpha) = 2f(\alpha)$ then find the value of α .

Q.2 Write the set $\{x: x \text{ is a prime natural number which divides } 5151 \text{ in tabular form}\}$

Q.3 How many words can be formed out of letters of the word. TRIANGLE ? How many of these will begin with T and end with E ?

Q.4 Find the third term in the expansion $(3x + 5y)^6$

Q.5 Identify the quantifier in the following statement “ there exists a real number whose square is not positive “ and write its negation.

Q.6 Find the component statement of the following compound statement. “ 100 is divisible by 3, 11 and 5 “ and check whether it is true or false.

Find the mode and median of the following data 2,3,2,4,6,4,5,4,3,1,4,6.

Q.8 Let $f(x) = ax + b$ if $f(2) = 8$ and $f(3) = 17$ find a and b .

Q.9 Let $A = \{ 2, 3, 4, 5, 6 \}$. Let R be the relation on A defined by the rule $x R y$ iff x divides y . Find R as a subset of $A \times A$.

Q.10 Write the contra positive and converse of the following statement.

“ Something is cold implies that it has low temperature “.

Section B

Q.11 Find the equation of the line passing through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axes.

Q.12 Find the ratio in which the YZ – plane divide the line segment formed by joining the point $(-2, 4, 7)$ and $(3, -5, 8)$. Also find the coordinates of the point of intersection.

Q.13 If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary. How many words are there in this list before the first word starting with E ?

OR

In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together ?

Q.14 If $a + ib = (x + i)^2/(2x^2 + 1)$ prove that $a^2 + b^2 = (x^2 + 1)^2/(2x^2 + 1)^2$

OR

If $(x + iy)^3 = u + iv$, then show that $u/x + v/y = 4(x^2 - y^2)$

Let $f = \{ (x, x^2/(1 + x^2)) , x \in \mathbb{R} \}$ be a function from \mathbb{R} into \mathbb{R} . Determine the range of f .

Q.16 A box contains 10 red marbles 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box. What is the probability that

(i) all will be blue.

(ii) at least one will be green.

Q.17 Find the term independent of x in the expansion of $(x^2/6 - 3/x^3)^{10}$, $x \neq 0$.

Q.18 If A, B, C are any three sets. Prove that $A - (B \cup C) = (A - B) \cap (A - C)$

OR

For any two sets A and B . Show that

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

Q.19 Prove that $\cos 7x + \cos 5x + \cos 3x + \cos x = 4 \cos x \cos 2x \cos 4x$

OR

Find the general solution of the following equation :

$$\sec^2 2x = 1 - \tan 2x$$

Q.20 Find the derivative of $\sin 2x$ from first principle.

OR

Find the derivative of $(x - 1)(x - 2)$ from first principle.

Q.21 Suppose that $f(x) = \sqrt{x^2 - 16}$ then find the domain of f .

Q.22 Find x and y if $\frac{x+i}{x-i} + \frac{x-i}{x+i} = i$.

Section C

Q.23 If the first and n th term of a G.P. are a and b respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$

OR

If p th, q th and r th terms of an A.P. are a, b, c respectively, show that

$$(q - r)a + (r - p)b + (p - q)c = 0$$

Q.24 (i) Find the equation of the circle passing through the points $(4, 1)$ and $(6, 5)$ and whose centre is on the line $4x + y = 16$.

(ii) Find eccentricity and Latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Q.25 Find $\sin x/2 \cos x/2$ and $\tan x/2$

If $\tan x = -4/3$, x in quadrant II

Q.26 Solve the following system of in equalities

$$4x + 3y \leq 60, y \leq 2x, x \leq 3, x, y \geq 0$$

Q.27 Prove the following by the principle of Mathematical Induction. For all n

$n \in \mathbb{N}$

$$1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+n)} = \frac{2n}{(n+1)}$$

OR

Prove by the principle of Mathematical Induction for all $n \in \mathbb{N}$ $3^{2n+2} - 8n - 9$ is divisible by 8.

Q.28 In an university, out of 100 students 15 offered Mathematics only; 12 offered statistics only; 8 offered only Physics; 40 offered Physics and Mathematics; 20 offered Physics and Statistics; 10 offered Mathematics and Statistics, 65 offered Physics. Find the number of students who

(i) offered Mathematics

(ii) offered Statistics

(iii) did not offer any of the above three subjects.

Q.29 Find the mean and variance for the following frequency distribution

Class 0-30 30-60 60-90 90-120 120-150 150-180 180-210

Frequency 2 3 5 10 3 5 2